

# Frontiers of particle accelerators: from sub-nuclear microscopes to medical instruments, an experience with accelerated beams

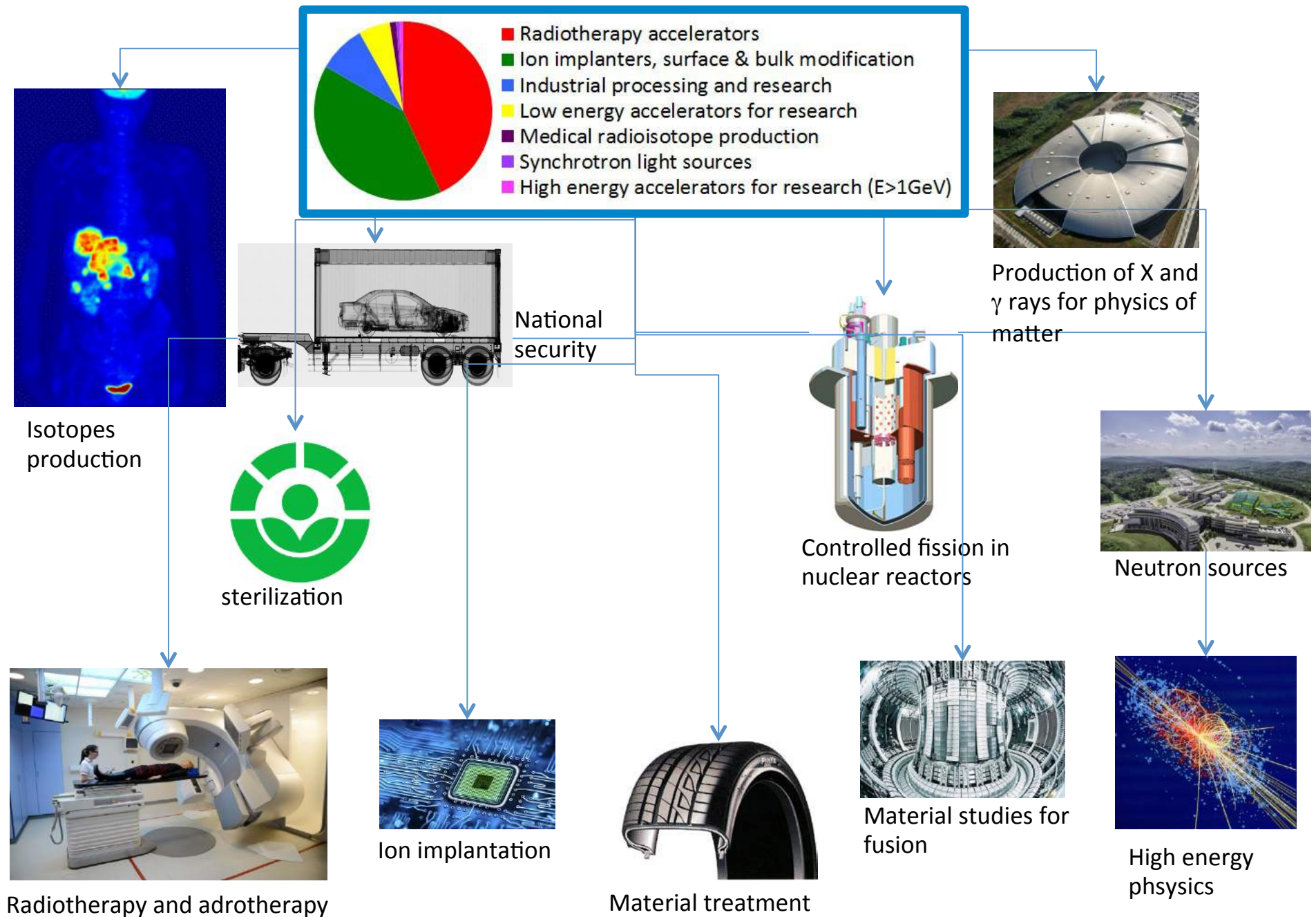
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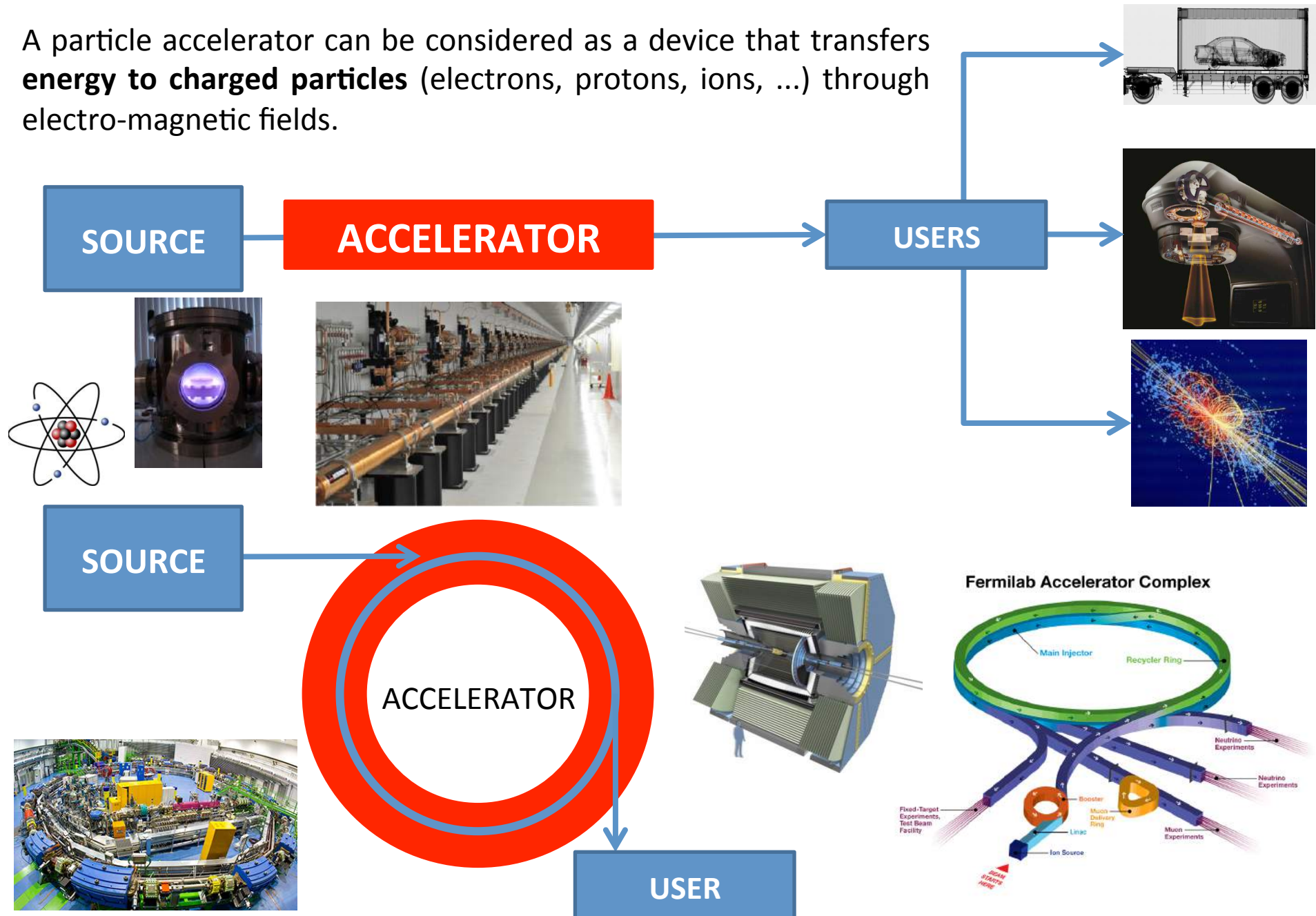
**Frascati, February 12 – 16, 2018**

# WHAT CAN PARTICLE ACCELERATORS DO?

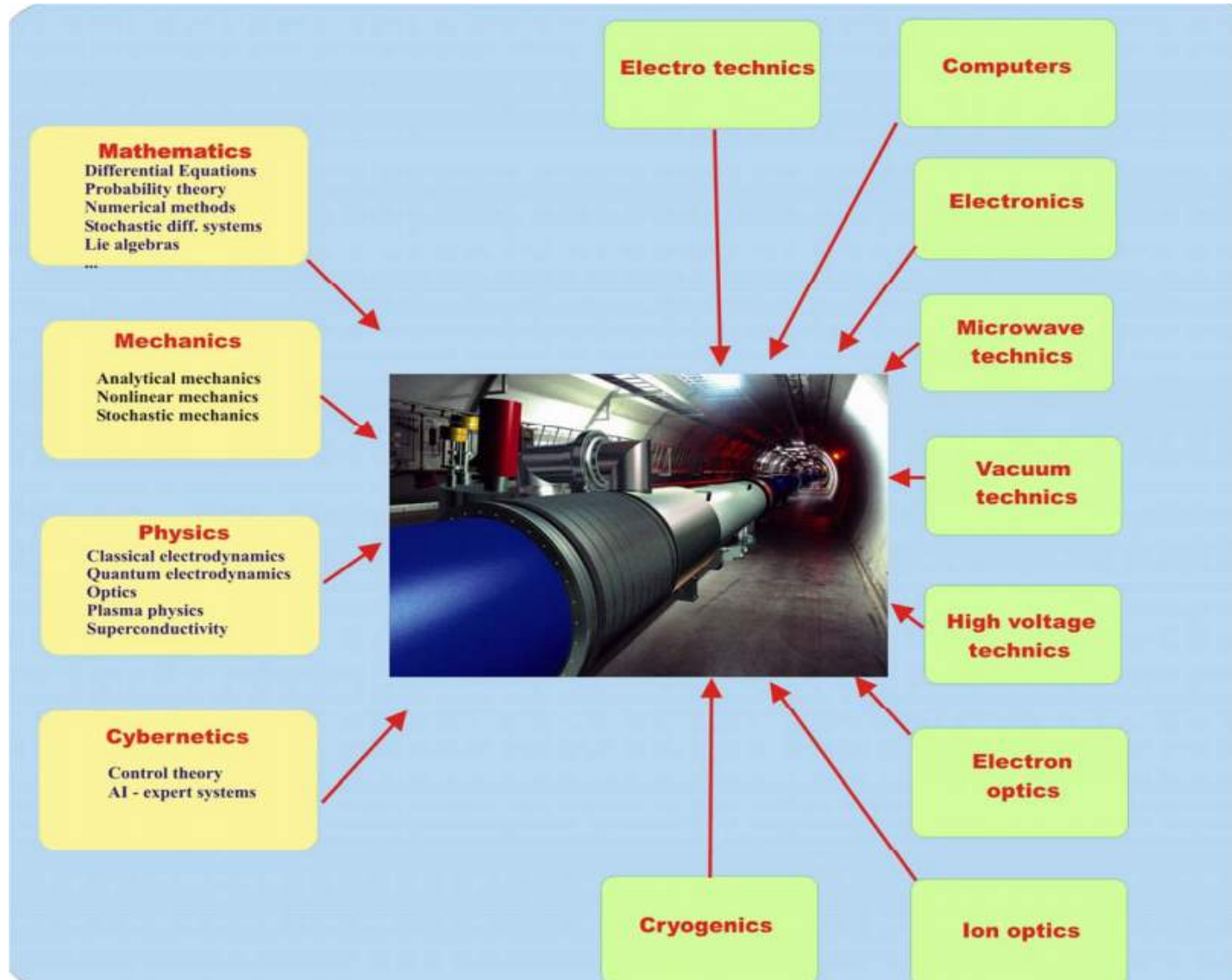


# HOW DO PARTICLE ACCELERATORS WORK?

A particle accelerator can be considered as a device that transfers **energy to charged particles** (electrons, protons, ions, ...) through electro-magnetic fields.



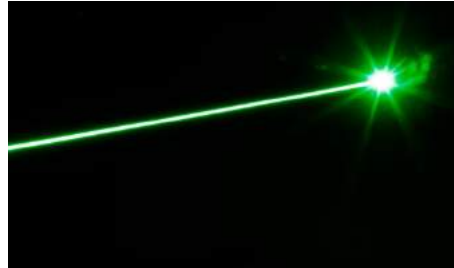
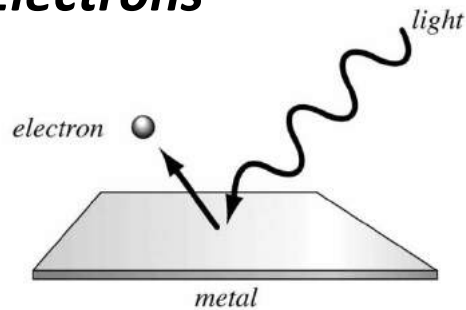
# INTERDISCIPLINARITY OF PHYSICS AND TECHNOLOGY OF PARTICLE ACCELERATOR



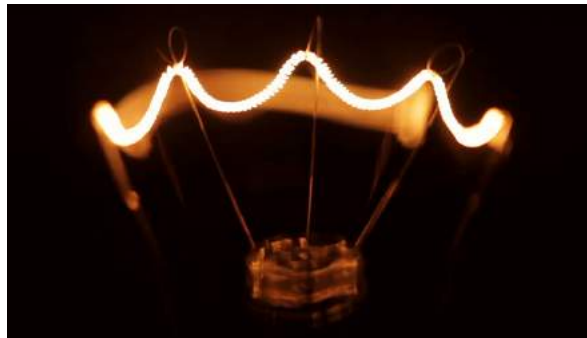
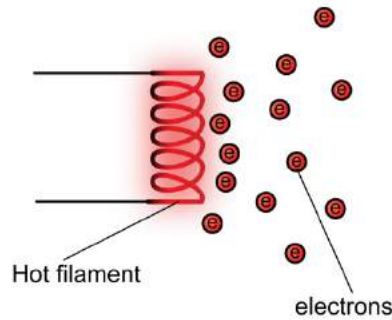


# PARTICLE ACCELERATOR SOURCES

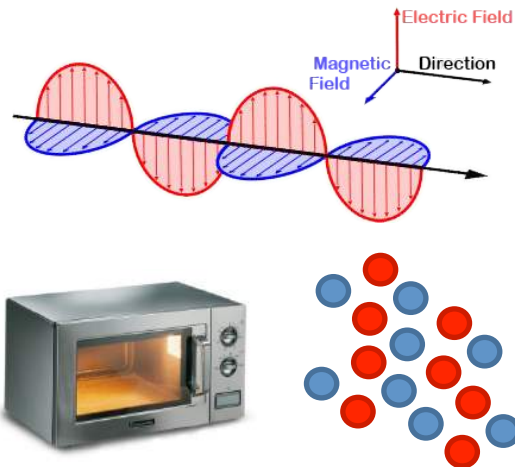
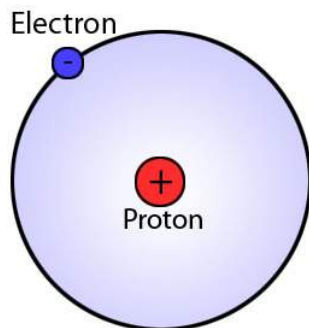
## Electrons



**Electrons** can be produced through laser light pulses that hit a metallic surface (**photo-electric effect**) or incandescent filaments (**thermionic effect**).



## Proton



**Protons** can be generated from hydrogen molecules that are transformed in **plasma**, through radiofrequency fields

# BASIC EQUATION FOR PARTICLE ACCELERATORS

Beams of charged particles are accelerated with the use of **electric fields** and are deflected, curved, and focused with the use of **magnetic fields**. The basic equation for the description of the acceleration and focusing processes is represented by the **Lorentz Force**.

$$\frac{d\vec{p}}{dt} = q \left( \vec{E} + \vec{v} \times \vec{B} \right)$$

$\vec{p}$  = momentum

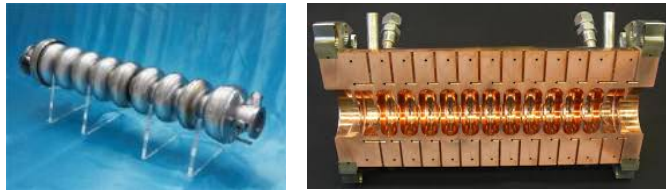
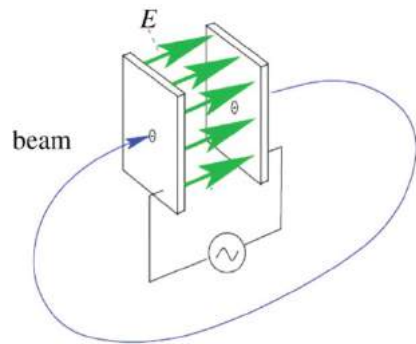
$m$  = mass

$\vec{v}$  = velocity

$q$  = charge

## ACCELERATION

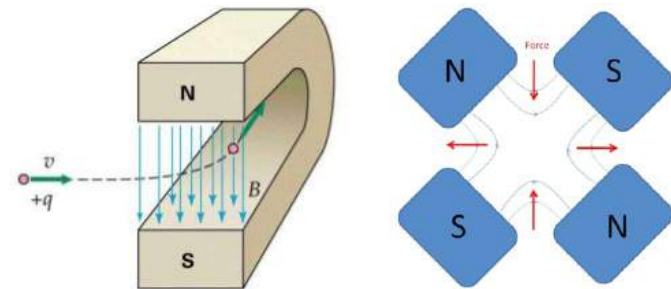
To accelerate, we need a force in the direction of motion



Longitudinal Dynamics

## BENDING AND FOCUSING

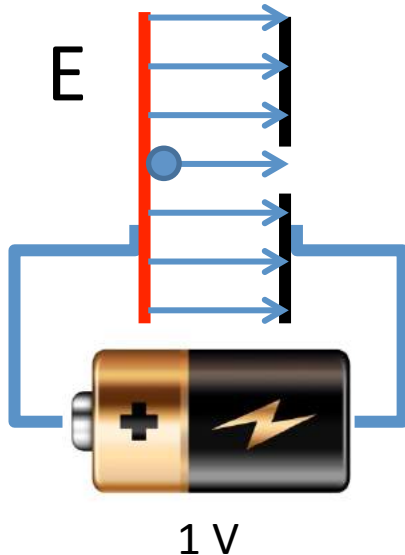
2<sup>nd</sup> term always perpendicular to motion => no energy gain



Transverse Dynamics

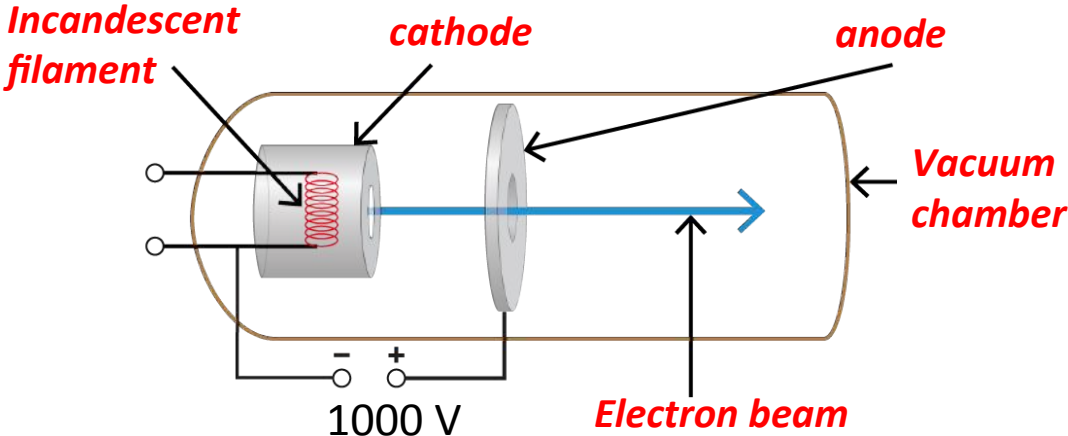
# PARTICLE ACCELERATION: ELECTRIC FIELD

Particles are accelerated through electric fields

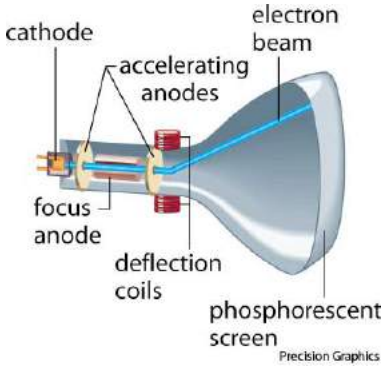


Energy gain  $\propto V$

$10^9 - 10^{10}$  V



220 V

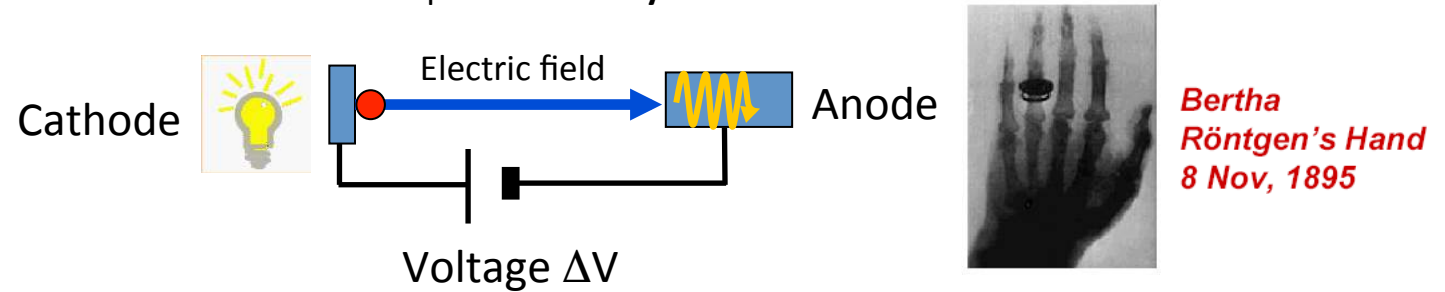


$10^5$  V



# ACCELERATION: SIMPLE CASE

The **first historical linear particle accelerator** was built by the Nobel prize Wilhelm Conrad Röntgen (1900). It consisted in a vacuum tube containing a cathode connected to the negative pole of a DC voltage generator. **Electrons emitted by the heated cathode** were accelerated while flowing to another electrode connected to the positive generator pole (anode). Collisions between the energetic electrons and the anode produced **X-rays**.



The **energy gained** by the electrons travelling from the cathode to the anode is equal to their charge multiplied the DC voltage between the two electrodes.

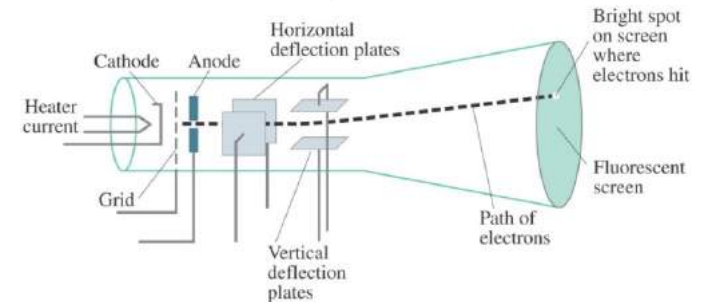
$$\frac{d\vec{p}}{dt} = q\vec{E} \Rightarrow \Delta E = q\Delta V$$

$\vec{p}$  = momentum

$q$  = charge

$E$  = energy

**Particle energies are typically expressed in electron-volt [eV]**, equal to the energy gained by 1 electron accelerated through an electrostatic potential of 1 volt:  
 1 eV =  $1.6 \times 10^{-19}$  J





# PARTICLE VELOCITY VS ENERGY: LIGHT AND HEAVY PARTICLES

Single particle

- rest mass  $m_0$
- rest energy  $E_0 (=m_0c^2)$
- total energy  $E$
- mass  $m$
- velocity  $v$
- momentum  $p (=mv)$
- Kinetic energy  $W=E-E_0$

Relativistic factor  
 $\beta=v/c (<1)$   
 Relativistic factor  
 $\gamma=E/E_0 (\geq 1)$

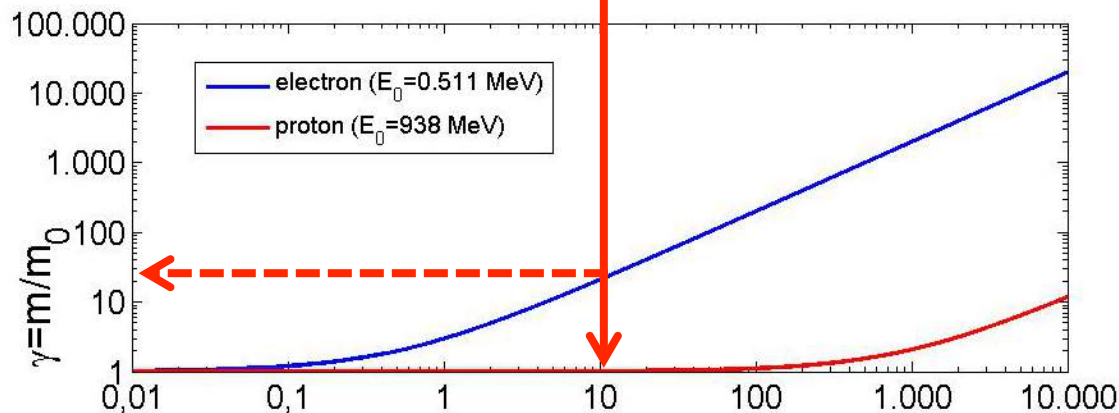
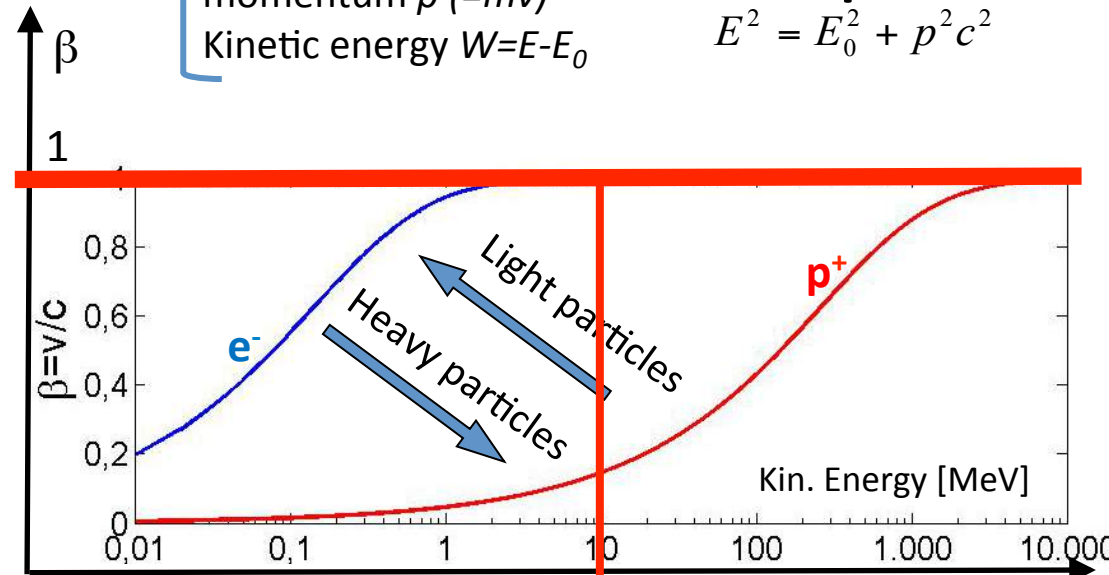
$$E^2 = E_0^2 + p^2 c^2$$

$$\beta = \sqrt{1 - 1/\gamma^2}$$

$$\gamma = 1/\sqrt{1 - \beta^2} \quad (m = \gamma m_0)$$

$$W = (\gamma - 1)m_0c^2 \approx \frac{1}{2}m_0v^2 \quad \text{if } \beta \ll 1$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \left(\frac{E_0}{E}\right)^2} = \sqrt{1 - \left(\frac{E_0}{E_0 + W}\right)^2}$$



⇒ **Light particles** (as **electrons**) are practically fully relativistic ( $\beta \approx 1$ ,  $\gamma \gg 1$ ) at relatively low energy and **reach a constant velocity** ( $\sim c$ ). The acceleration process occurs at constant particle velocity

⇒ **Heavy particles** (protons and ions) are typically weakly relativistic and **reach a constant velocity only at very high energy**. The velocity changes a lot during acceleration process.

⇒ This implies **important differences** in the technical characteristics of the **accelerating structures**. In particular for protons and ions we need different types of accelerating structures, **optimized for different velocities** and/or the accelerating structure has to vary its geometry to take into account the velocity variation.

# ELECTROSTATIC ACCELERATORS

To increase the achievable maximum energy, Van de Graaff invented an electrostatic generator based on a **dielectric belt** transporting positive charges to an isolated electrode hosting an **ion source**. The positive ions generated in a large positive potential were accelerated toward ground by the static electric field.

## LIMITS OF ELECTROSTATIC ACCELERATORS

DC voltage as large as **~10 MV** can be obtained ( $E \sim 10$  MeV). The main limit in the achievable voltage is the **breakdown** due to **insulation** problems.

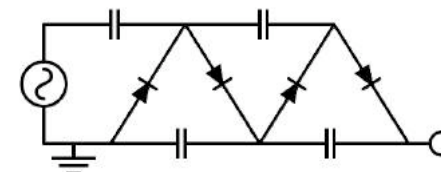
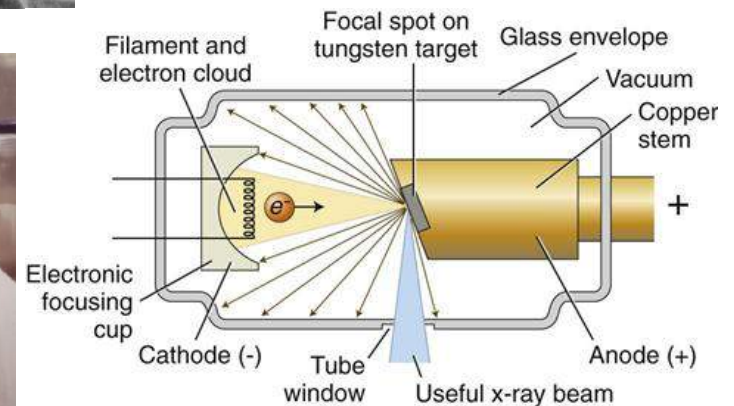
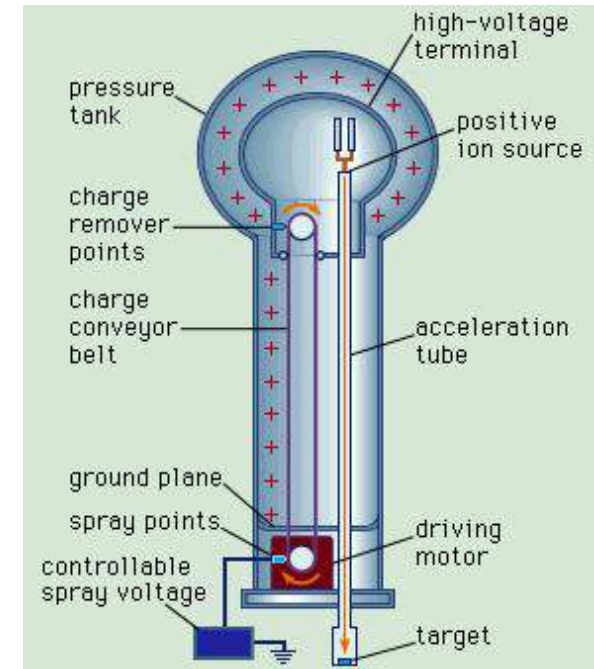
## APPLICATIONS OF DC ACCELERATORS

DC particle accelerators are in operation worldwide, typically at  $V < 15$  MV ( $E_{\max} = 15$  MeV),  $I < 100$  mA.

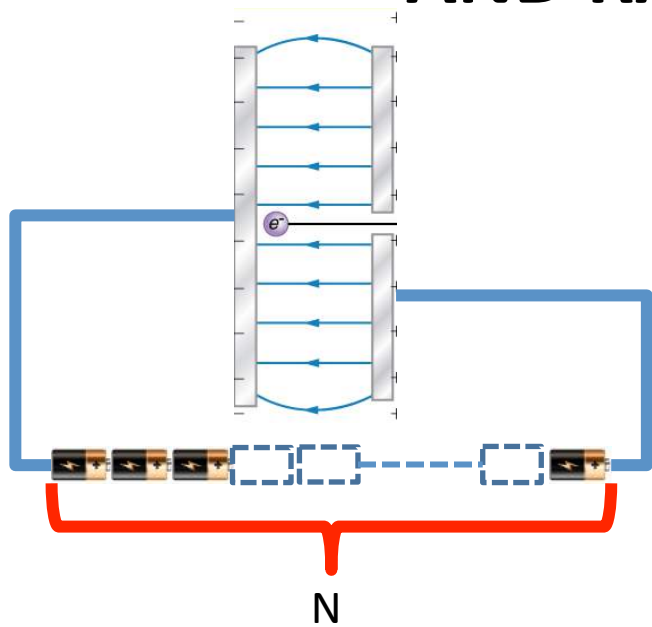
They are used for:

- ⇒ *material analysis*
- ⇒ *X-ray production,*
- ⇒ *ion implantation for semiconductors*
- ⇒ *first stage of acceleration (particle sources)*

750 kV Cockcroft-Walton  
Linac2 injector at CERN from 1978  
to 1992



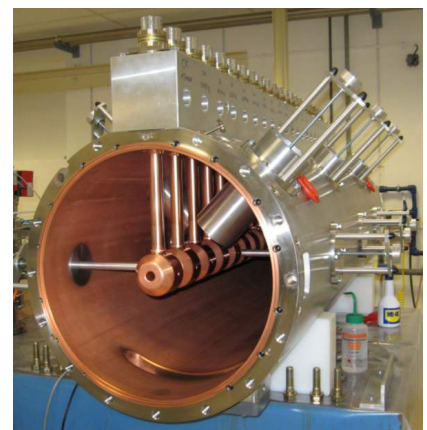
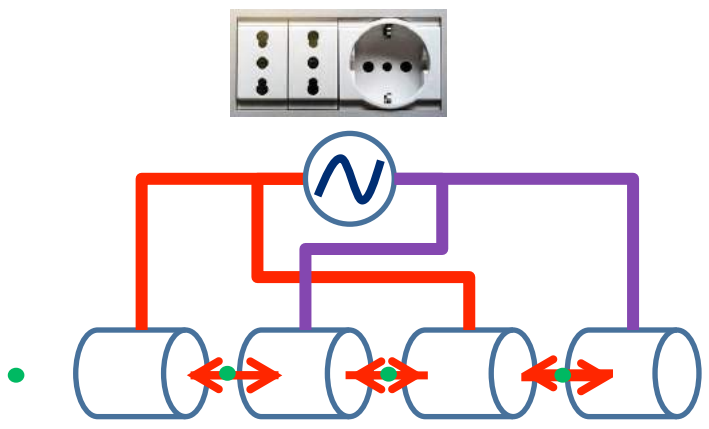
# ACCELERATION OF PARTICLES WITH ELECTROSTATIC AND RADIOFREQUENCY FIELDS



$V < 10\text{MV}$  ( $10^7\text{ V}$ )



In these structures the maximum energy is theoretically limited only by the maximum **length of the accelerator**

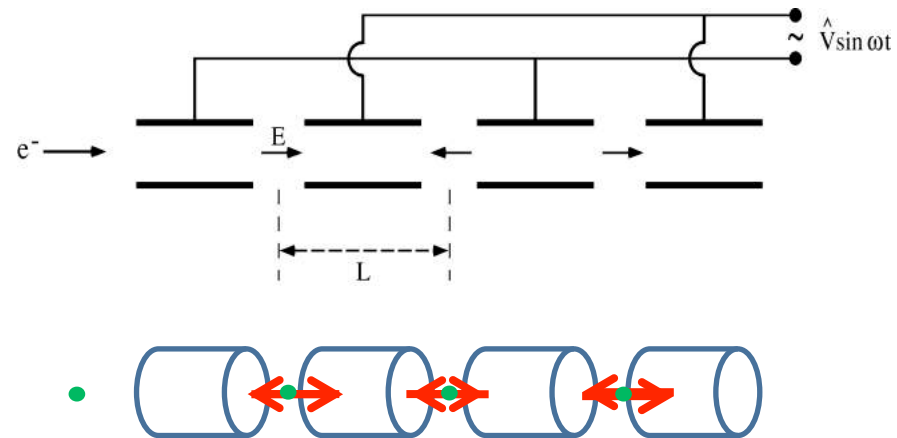


One can obtain up to 100 MV/m



# RF ACCELERATORS : WIDERÖE “DRIFT TUBE LINAC” (DTL)

Basic idea: the particles are accelerated by the electric field in the gap between electrodes connected alternatively to the poles of an AC generator. This original idea of **Ising** (1924) was implemented by **Wideroe** (1927) who applied a sine-wave voltage to a sequence of **drift tubes**. The particles **do not experience any force while travelling inside the tubes** (equipotential regions) and are **accelerated across the gaps**. This kind of structure is called **Drift Tube LINAC (DTL)**.

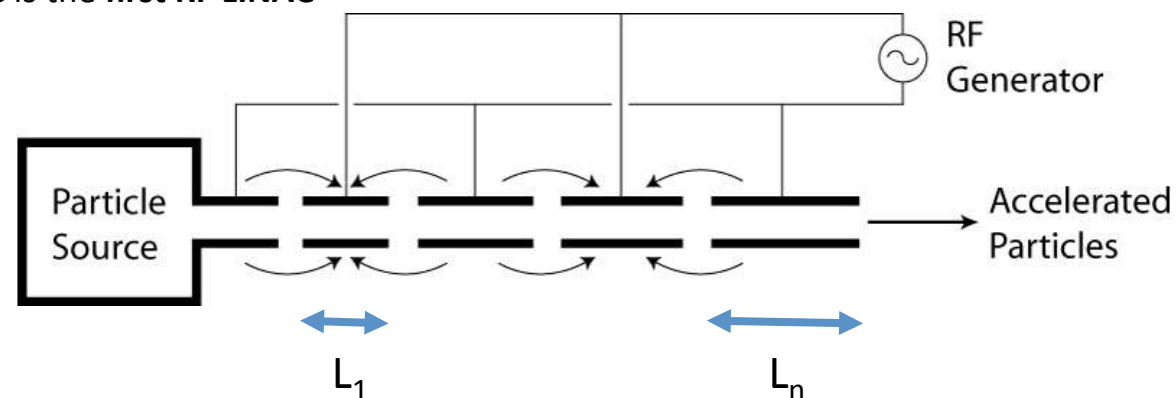


Rolf Wideroe

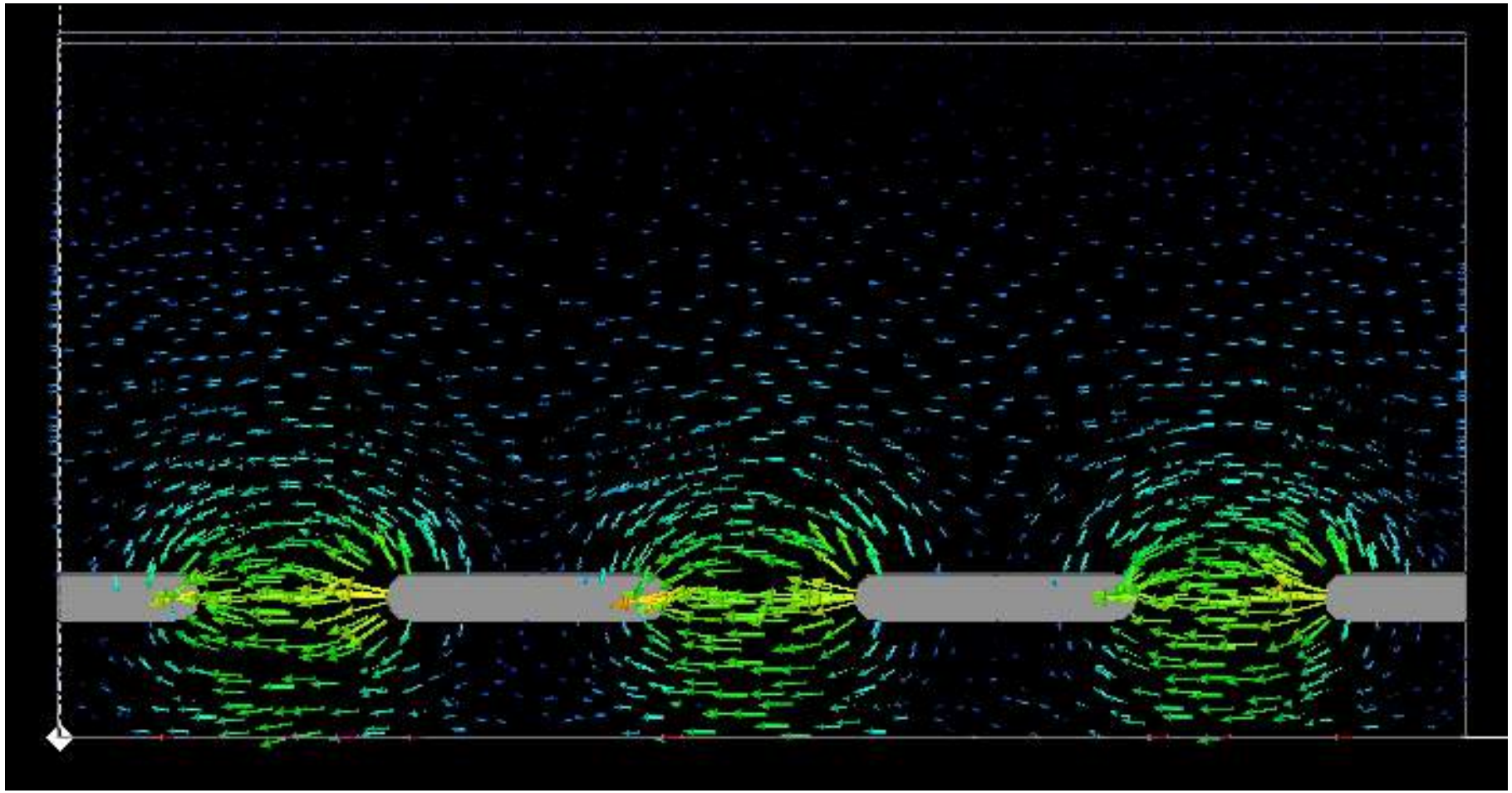
⇒ If the **length of the tubes** increases with the particle velocity during the acceleration such that the time of flight is kept constant and equal to half of the RF period, the particles are subject to a **synchronous accelerating voltage** and experience an energy gain of  $\Delta E = q\Delta V$  at each gap crossing.

⇒ In principle a single **RF generator** can be used to indefinitely accelerate a beam, **avoiding the breakdown limitation** affecting the electrostatic accelerators.

⇒ The Wideroe LINAC is the **first RF LINAC**

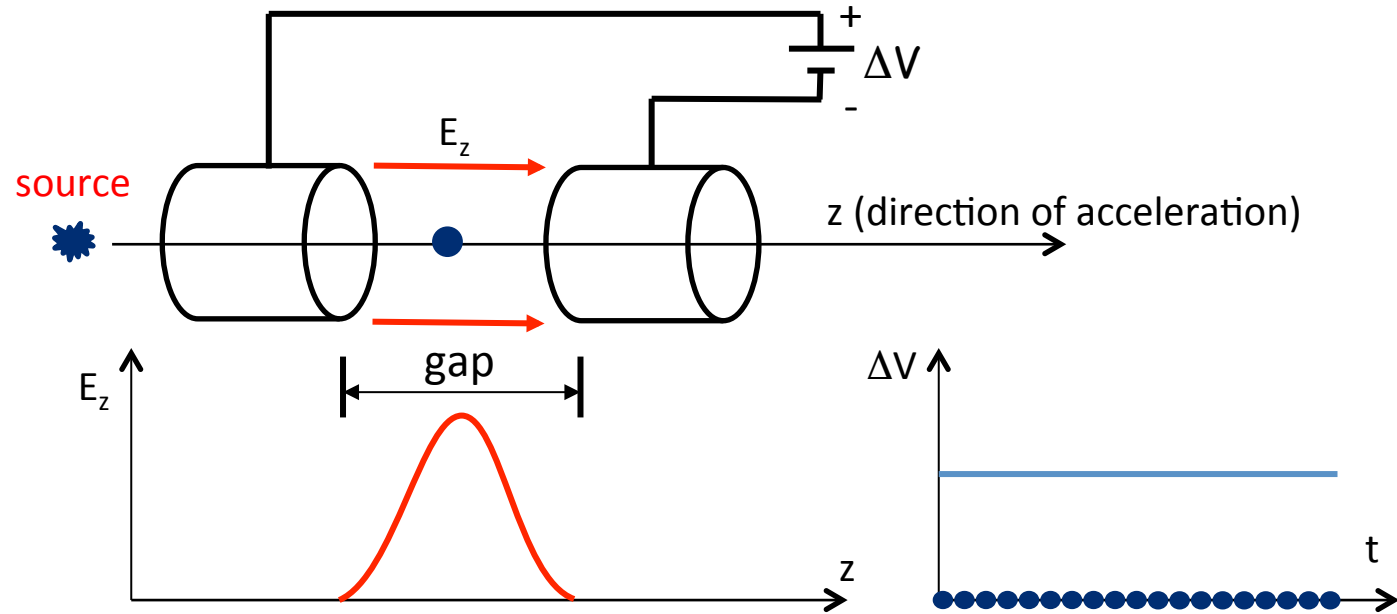






# ELECTROSTATIC ACCELERATION: CONTINUOUS BEAM

We consider the acceleration between two electrodes in DC.



$$E^2 = E_0^2 + p^2 c^2 \Rightarrow 2E dE = 2p dp c^2 \Rightarrow dE = v \frac{mc^2}{E} dp \Rightarrow dE = v dp$$

$$\frac{dp}{dt} = qE_z \Rightarrow v \frac{dp}{dz} = qE_z \Rightarrow \boxed{\frac{dE}{dz} = qE_z} \quad \left( \text{and also } \frac{dW}{dz} = qE_z \right) \quad W = E - E_0$$

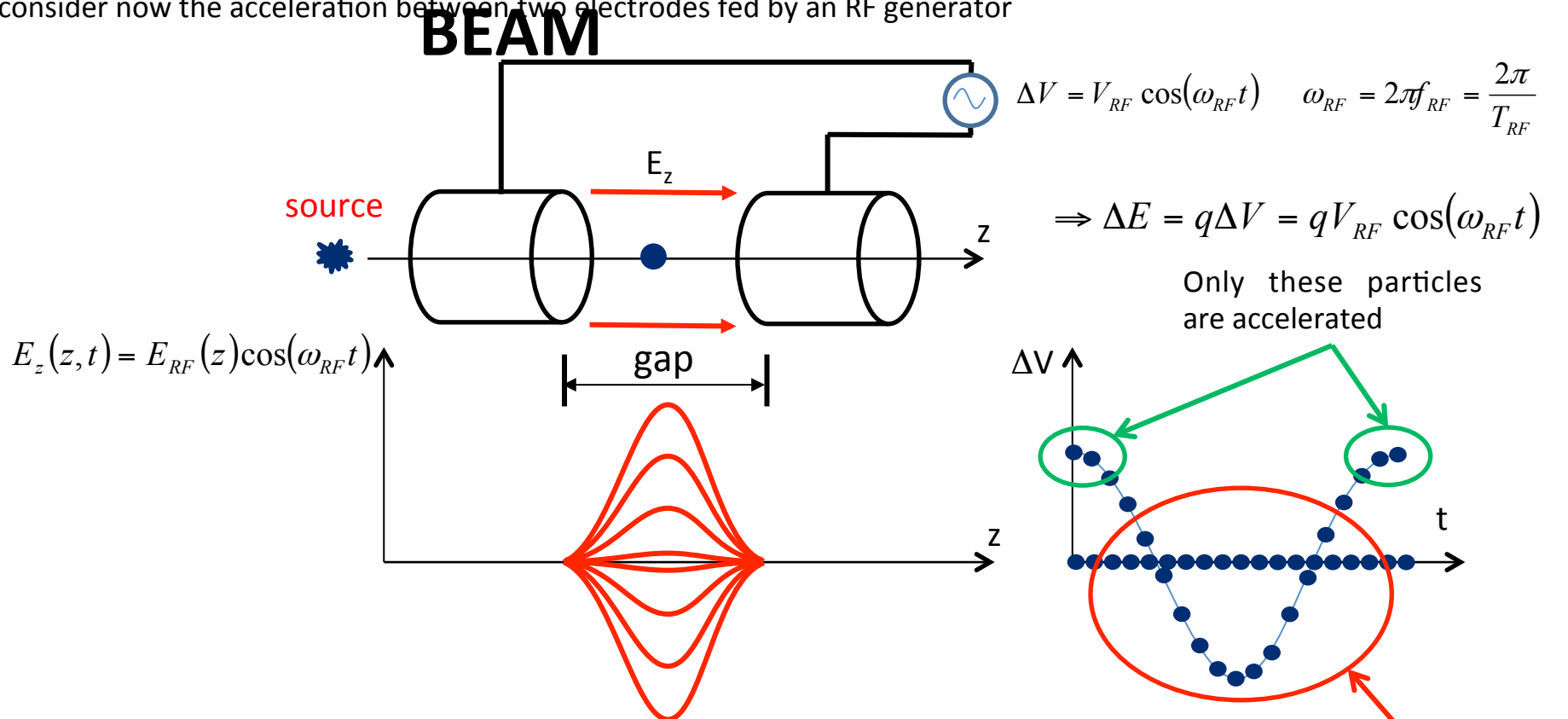
rate of energy gain per unit length

$$\Rightarrow \Delta E = \int_{gap} \frac{dE}{dz} dz = \int_{gap} qE_z dz \Rightarrow \boxed{\Delta E = q\Delta V}$$

energy gain per electrode

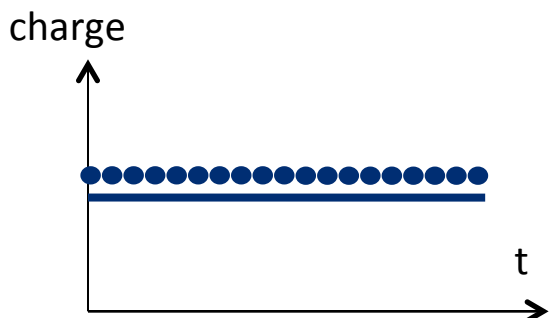
# RF ACCELERATION: BUNCHED

We consider now the acceleration between two electrodes fed by an RF generator

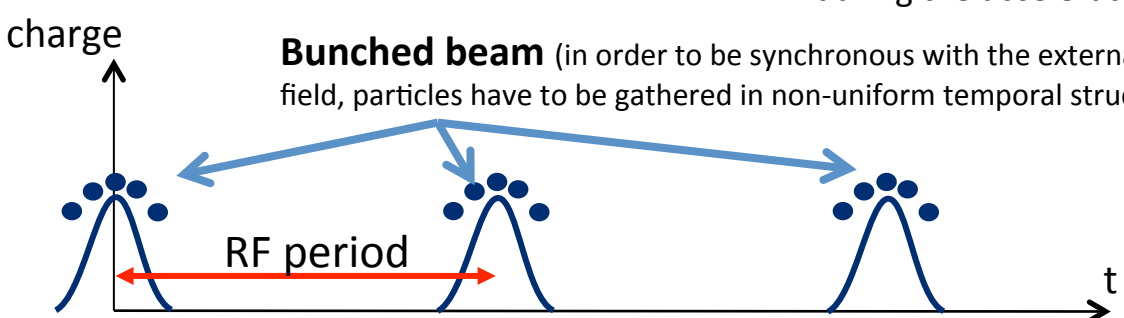


These particles are not accelerated and basically are lost during the acceleration process

## DC acceleration



## RF acceleration



# ACCELERATION WITH HIGH RF FREQUENCIES: RF CAVITIES

There are two important **consequences** of the previous obtained formulae:

$$L_n = \frac{1}{2} \beta_n \lambda_{RF}$$



The condition  $L_n \ll \lambda_{RF}$  (necessary to model the tube as an **equipotential region**) requires  $\beta \ll 1$ .  $\Rightarrow$  The Wideröe technique can **not be applied to relativistic particles**.

$$\frac{\Delta E}{\Delta L} = \frac{qV_{RF}}{L_n} = qE_{RF} = \frac{2qV_{RF}}{\lambda_{RF} \beta_n}$$



Moreover when particles get high velocities the drift spaces get longer and one loses on the efficiency. The **average accelerating gradient ( $E_{RF}$  [V/m]) increase pushes towards small  $\lambda_{RF}$**  (high frequencies).

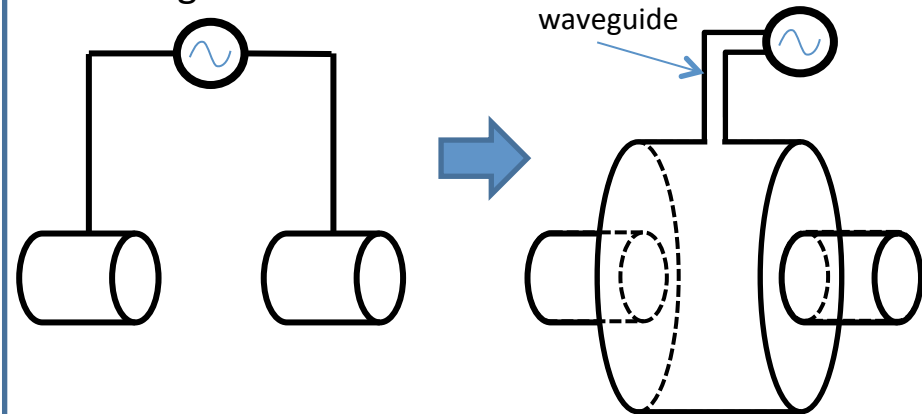
High frequency high power **sources** became available after the **2<sup>nd</sup> world war** pushed by **military** technology needs (such as **radar**). However, the concept of equipotential DT can not be applied at small  $\lambda_{RF}$  and the **power lost by radiation is proportional to the RF frequency**.

As a **consequence** we must consider **accelerating structures different from drift tubes**.



$\Rightarrow$  The solution consists of **enclosing the system in a cavity** which resonant frequency matches the RF generator frequency.

$\Rightarrow$  Each cavity can be independently powered from the RF generator





# RF CAVITIES

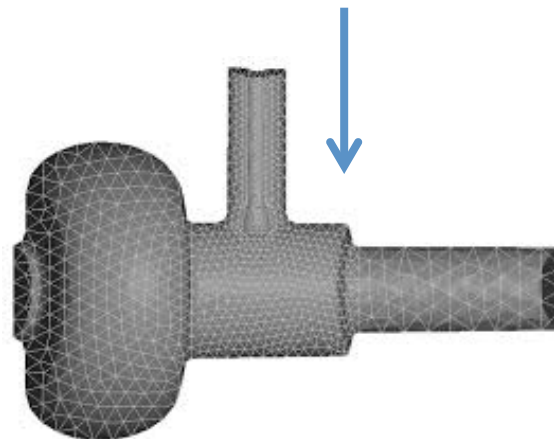
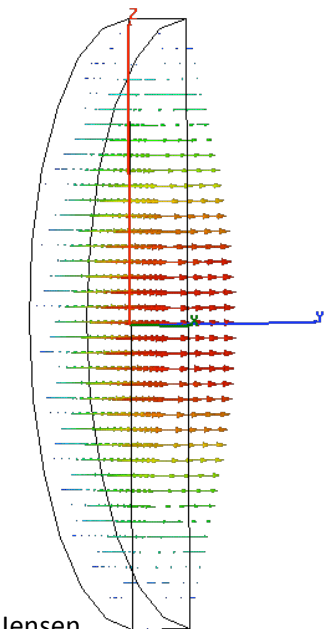
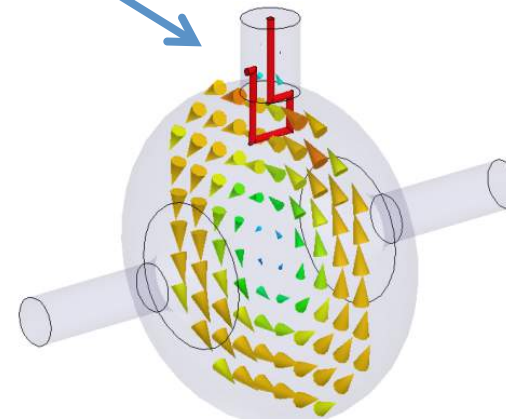
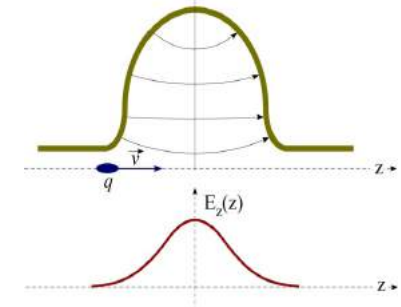
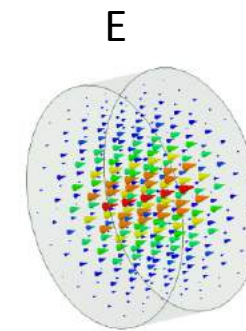
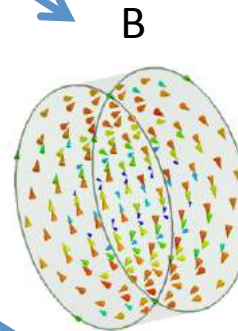
⇒ High frequency RF accelerating fields are confined in **cavities**.

⇒ The cavities are **metallic closed volumes** where the e.m fields has a particular spatial configuration (**resonant modes**) whose components, including the accelerating field  $E_z$ , oscillate at some specific frequencies  $f_{RF}$  (resonant frequency) characteristic of the mode.

⇒ The modes are excited by **RF generators** that are **coupled to the cavities** through waveguides, coaxial cables, etc...

⇒ The resonant modes are called **Standing Wave (SW) modes** (spatial fixed configuration, oscillating in time).

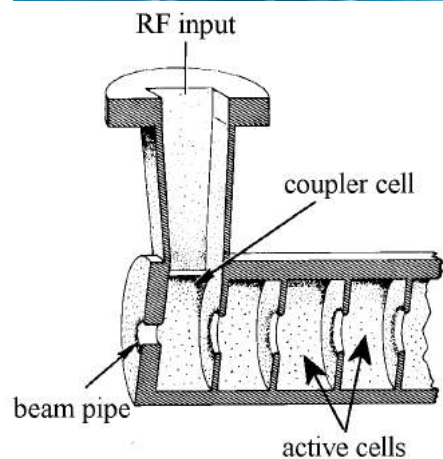
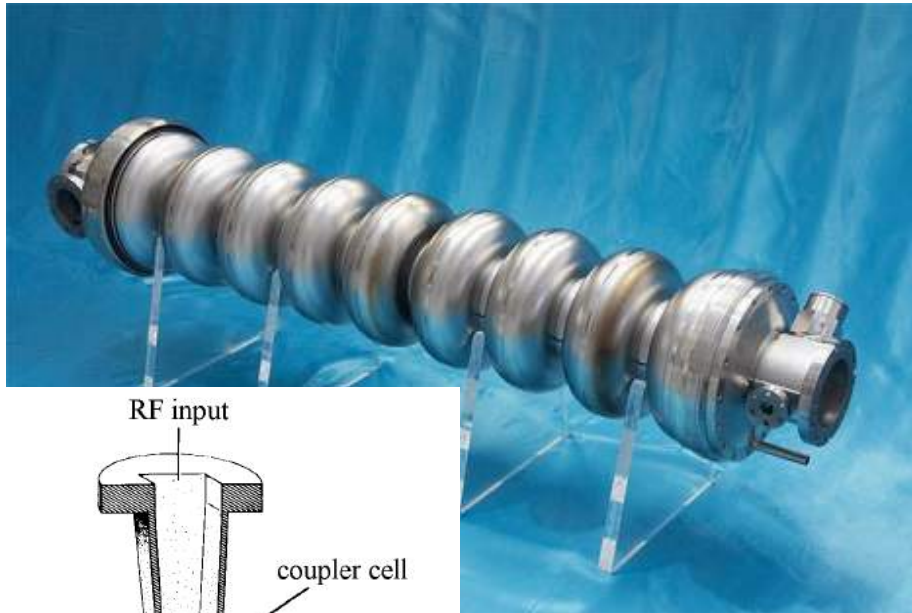
⇒ The spatial and temporal field profiles in a cavity have to be computed (analytically or numerically) **by solving the Maxwell equations** with the proper boundary conditions.



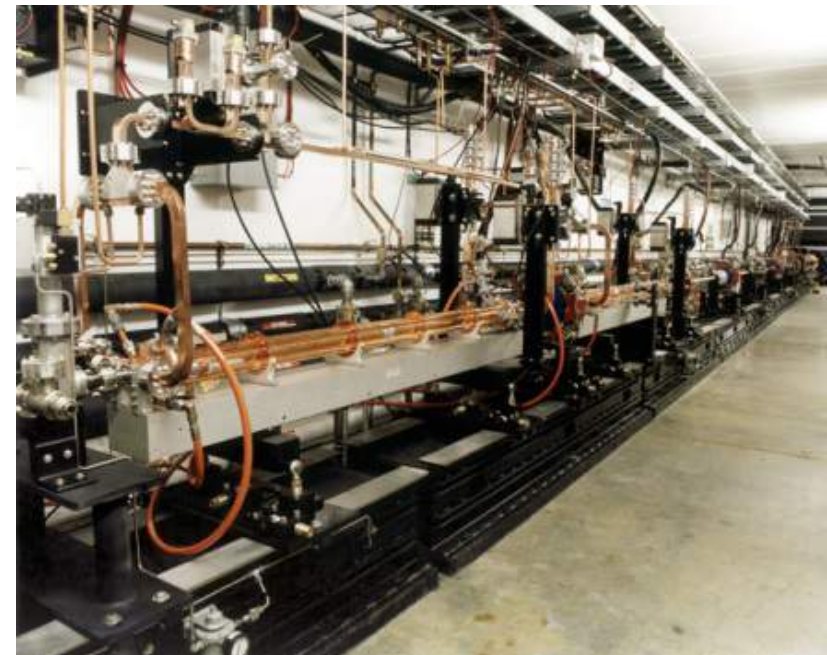
Courtesy E. Jensen

# MULTI-CELL CAVITIES

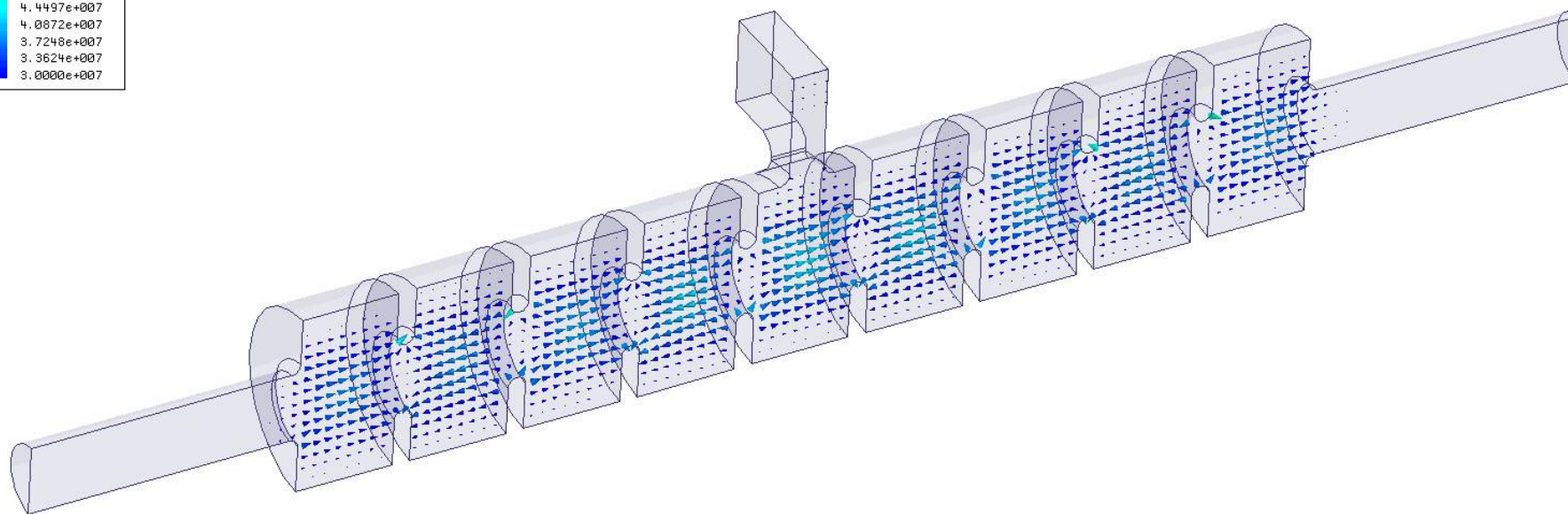
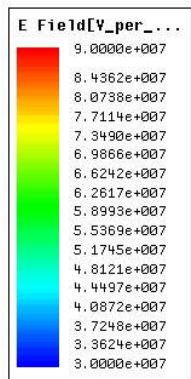
The **resonant cavities** are usually grouped in **multi-cell structures**. This choice is motivated by reasons of efficiency and compactness. In a multi-cell structure a single RF coupler is sufficient to excite the field. This implies the use of a reduced number of high-power RF sources, to the benefit of **simplicity and cost** of the accelerator. The coupling between the cells is achieved through irises in each cell and/or through apertures specifically made between the cells (coupling slots).



There are both cavities that operate at **room temperature** (typically in copper) and **superconducting** cavities that operate at few K. The average accelerating gradients that can typically be obtained are of the order of **few 10 MV/m up to more than 100 MV/m**.



The working frequencies can go from MHz to ten of GHz depending on the applications

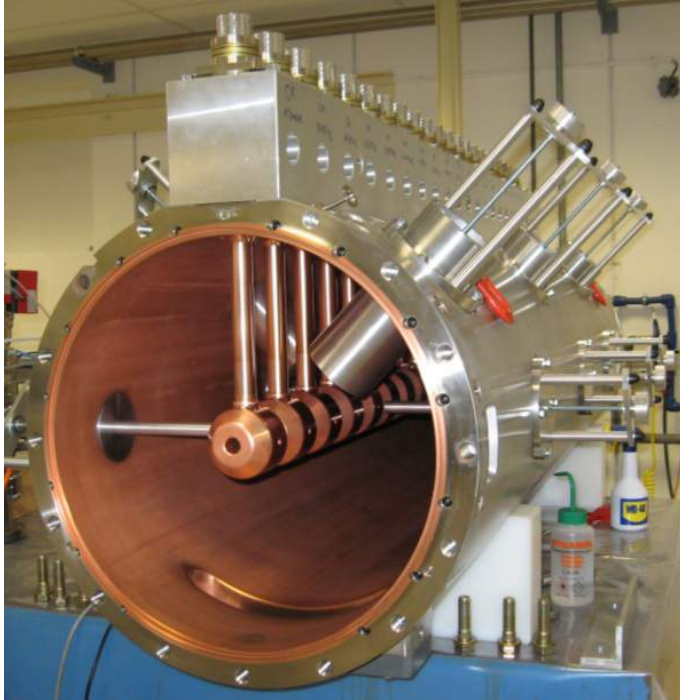




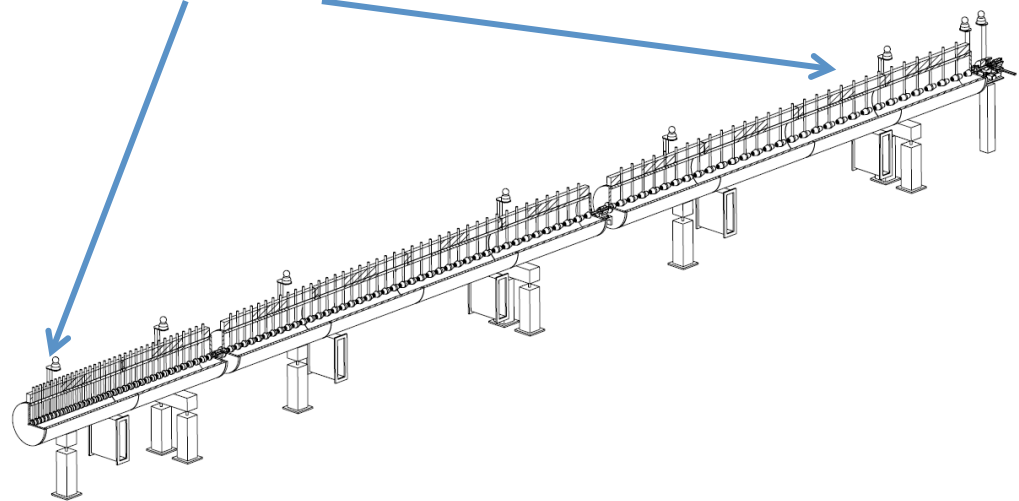
# ALVAREZ STRUCTURES: EXAMPLES



CERN LINAC 2 tank 1:  
200 MHz 7 m x 3 tanks, 1 m  
diameter, final energy 50 MeV.



CERN LINAC 4: 352 MHz frequency, Tank diameter 500 mm, 3 resonators (tanks), Length 19 m, 120 Drift Tubes, Energy: 3 MeV to 50 MeV,  $\beta=0.08$  to 0.31  $\rightarrow$  cell length from 68mm to 264mm.





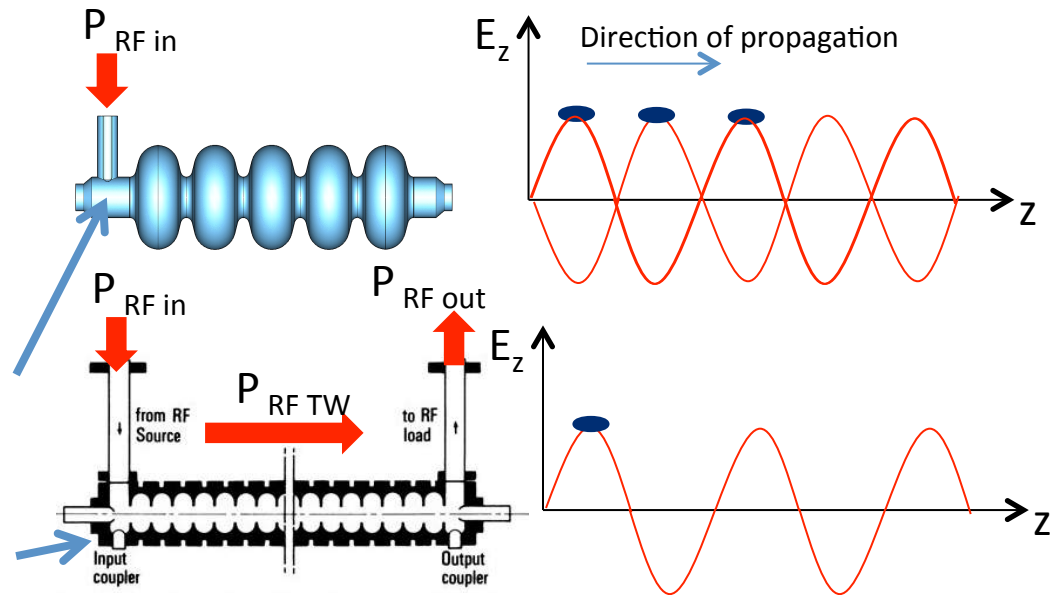
# SW AND TW ACCELERATING CAVITIES

$$\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

To accelerate charged particles, the RF wave must have an **electric field along the direction of propagation of the particle**. There are basically two possibilities:

1-Using **standing wave (SW)** TM<sub>010</sub>-like modes in a **resonant cavity** (or multiple resonant cavities) in which the beam is synchronous with the resonating field;

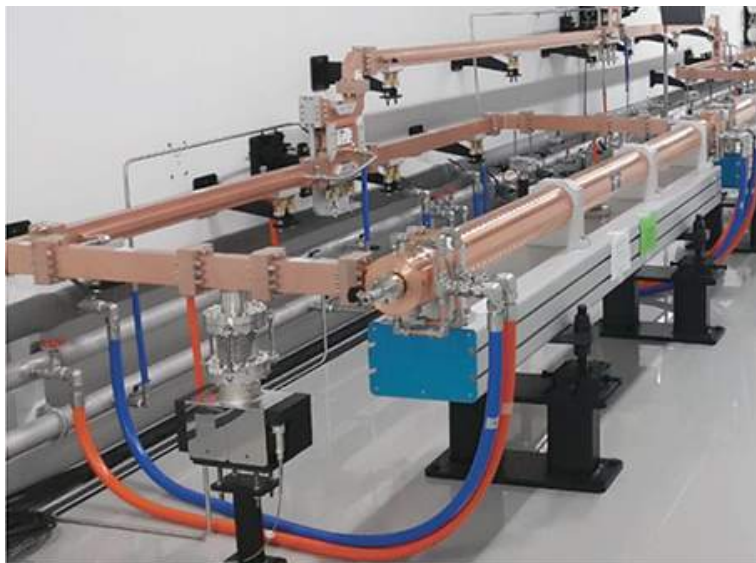
2-Using a **travelling wave (TW) disk loaded** structure operating on the TM<sub>01</sub>-like mode in which the RF wave is co-propagating with the beam with a phase velocity equal to the beam velocity ( $c$  for  $e^-$ ).



⇒The structures are powered by RF generators (typically **klystrons**).

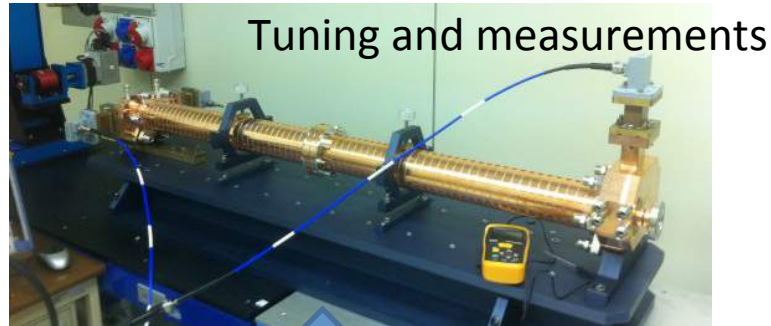
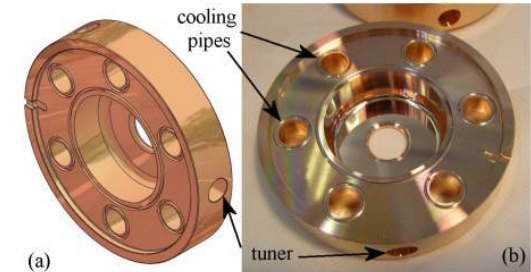
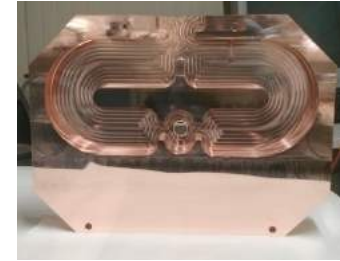
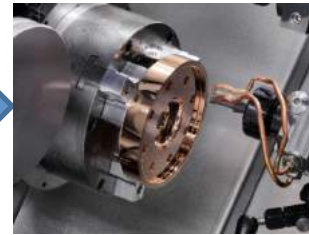
⇒The cavities (and the related LINAC technology) can be of different material:

- copper for **normal conducting (NC, typically TW)** cavities;
- Niobium for **superconducting cavities (SC, typically SW)**;

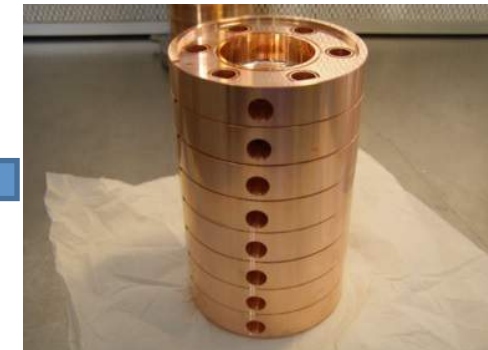
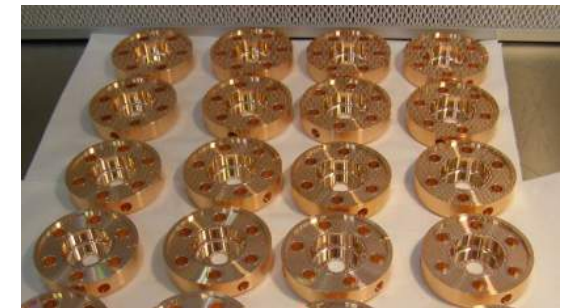


# EXAMPLE FABRICATION PROCESS: NC TW STRUCTURES

The cells and couplers are fabricated with milling machines and lathes starting from **OFHC forged or laminated copper** with precisions that can be of the order of few  $\mu\text{m}$  and surface roughness  $<50\text{ nm}$ .



The cells are then piled up and **brazed** together in vacuum or hydrogen furnace using different alloys at different temperatures (700-1000 C) and/or in different steps.



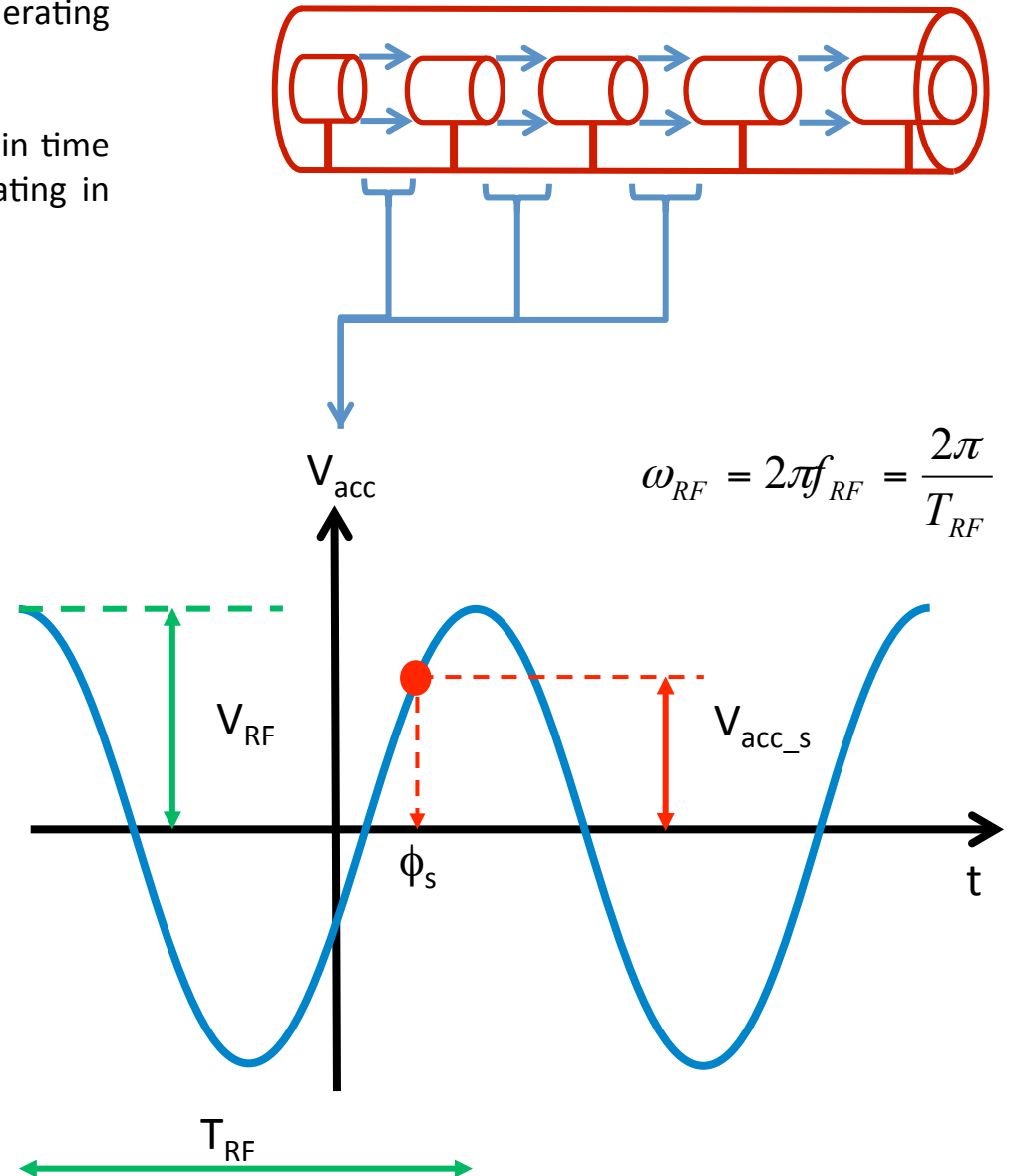
# LONGITUDINAL DYNAMICS: PARTICLE OSCILLATIONS IN LONGITUDINAL PLANE AROUND THE SYNCHRONOUS ONE

⇒ Let us consider a **SW linac structure** made by accelerating **gaps** (like in DTL) or **cavities**.

⇒ In **each gap** we have an **accelerating field** oscillating in time and an integrated accelerating voltage ( $V_{acc}$ ) still oscillating in time than can be expressed as:

$$V_{acc} = V_{RF} \cos(\omega_{RF}t + \theta)$$

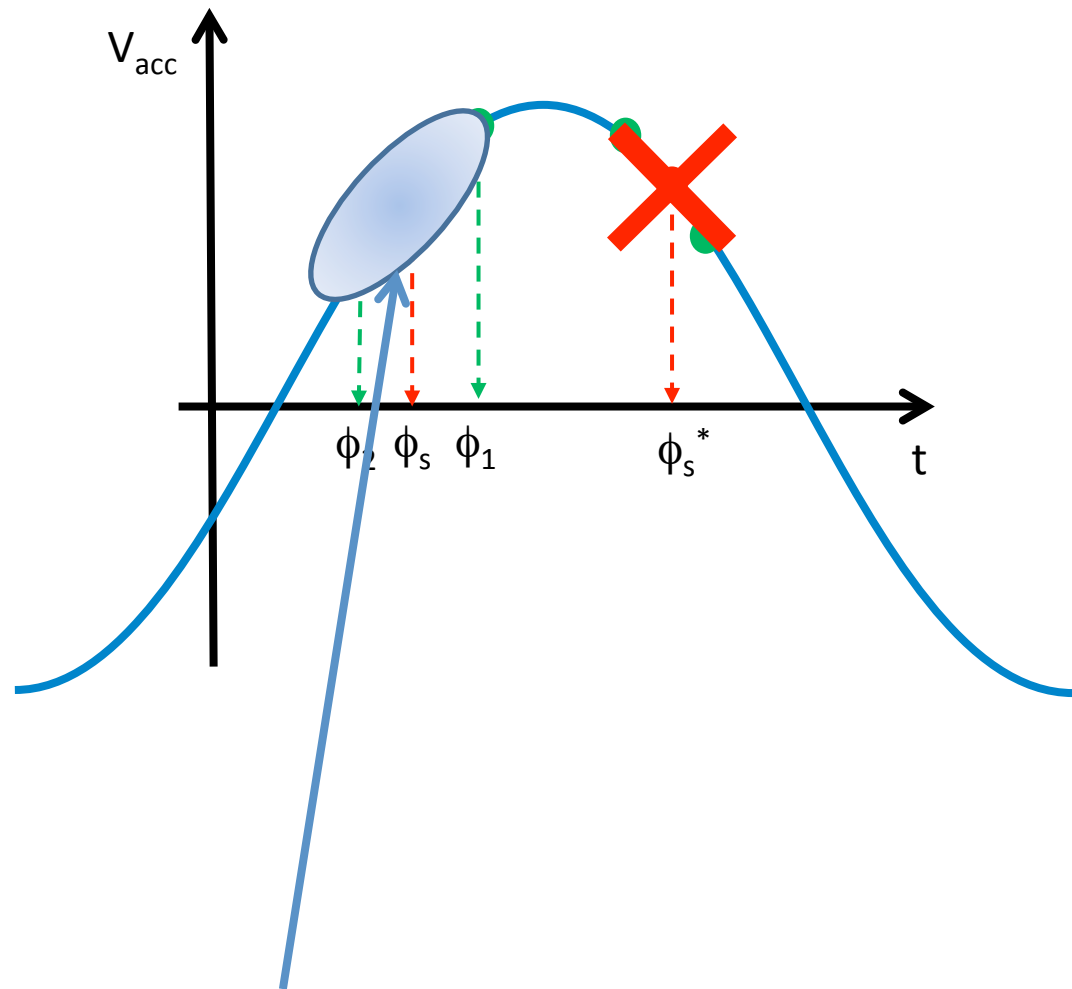
⇒ Let's assume that the **“perfect” synchronism condition is fulfilled for a phase  $\phi_s$**  (called **synchronous phase**). This means that a particle (called **synchronous particle**) entering in a gap with a phase  $\phi_s$  ( $\phi_s = \omega_{RF}t_s$ ) with respect to the RF voltage receive a **energy gain** (and a consequent change in velocity) that allow entering in the subsequent gap with the **same phase  $\phi_s$**  and so on.



# PRINCIPLE OF PHASE STABILITY

⇒ Let us consider now the first synchronous phase  $\phi_s$  (on the positive slope of the RF voltage). If we consider **another particle** “near” to the synchronous one **that arrives later in the gap** ( $t_1 > t_s$ ,  $\phi_1 > \phi_s$ ), it will see a higher voltage, it will gain a higher energy and a higher velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be shorter, partially **compensating its initial delay**.

⇒ **Similarly** if we consider another particle “near” to the synchronous one that arrives before in the gap ( $t_1 < t_s$ ,  $\phi_1 < \phi_s$ ), it will see a smaller voltage, it will gain a smaller energy and a smaller velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be longer, compensating the initial advantage.



All particles are “stably” grouped and accelerated around the synchr. one



# LORENTZ FORCE: ACCELERATION AND FOCUSING

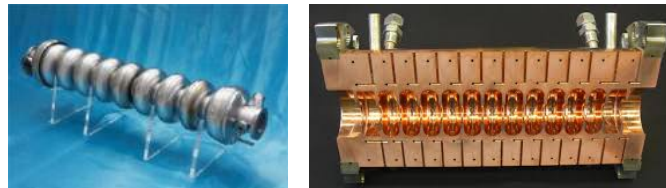
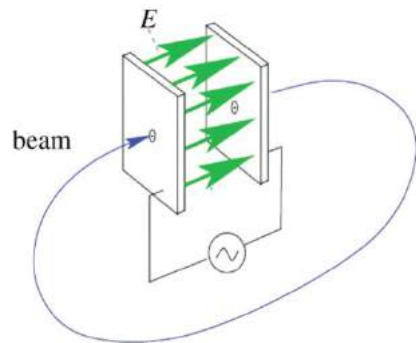
Particles are accelerated through electric field and are bended and focalized through magnetic field. The basic equation that describe the acceleration/bending /focusing processes is the **Lorentz Force**.

$\vec{p}$  = momentum  
 $m$  = mass  
 $\vec{v}$  = velocity  
 $q$  = charge

$$\frac{d\vec{p}}{dt} = q \left( \vec{E} + \vec{v} \times \vec{B} \right)$$

## ACCELERATION

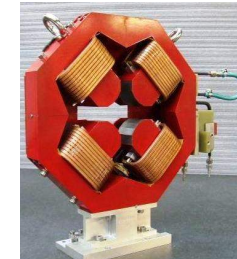
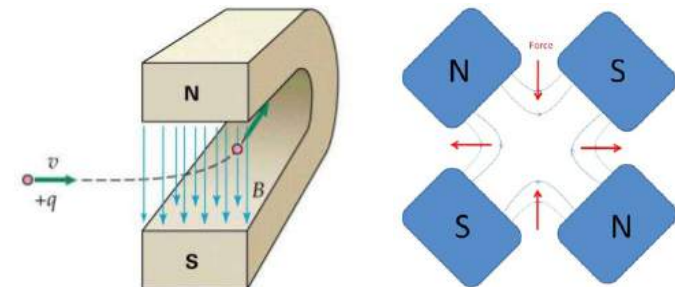
To accelerate, we need a force in the direction of motion



Longitudinal Dynamics

## BENDING AND FOCUSING

2<sup>nd</sup> term always perpendicular to motion => no energy gain



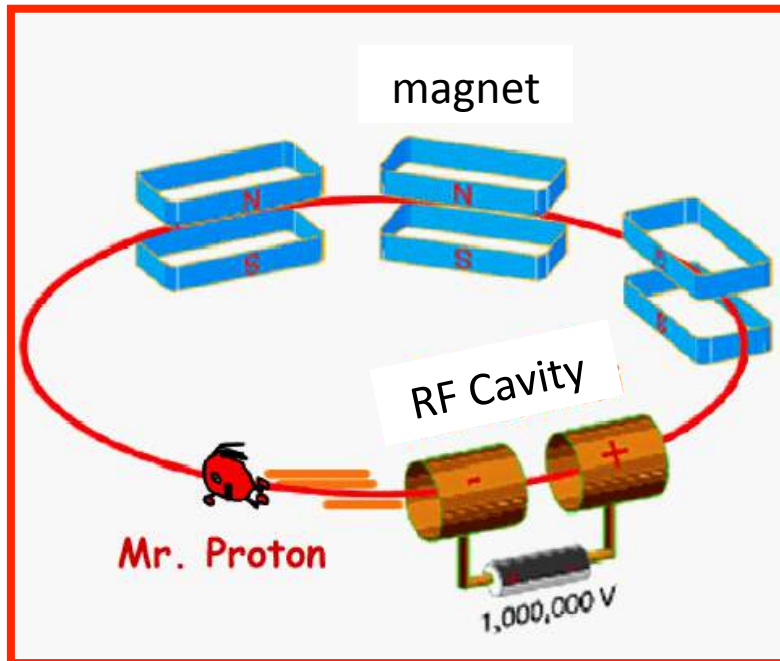
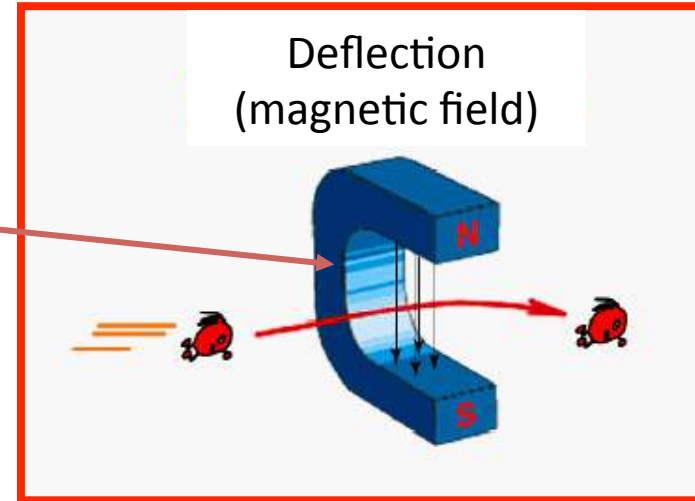
Transverse Dynamics

# MAGNETIC FIELD: DEFLECTION AND FOCUSING

With **magnetic fields** it is possible to **bend** charged particles (moving at speed  $v$ ) and it is possible to **focus** them keeping the particles confined inside in the vacuum chamber.

$$\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

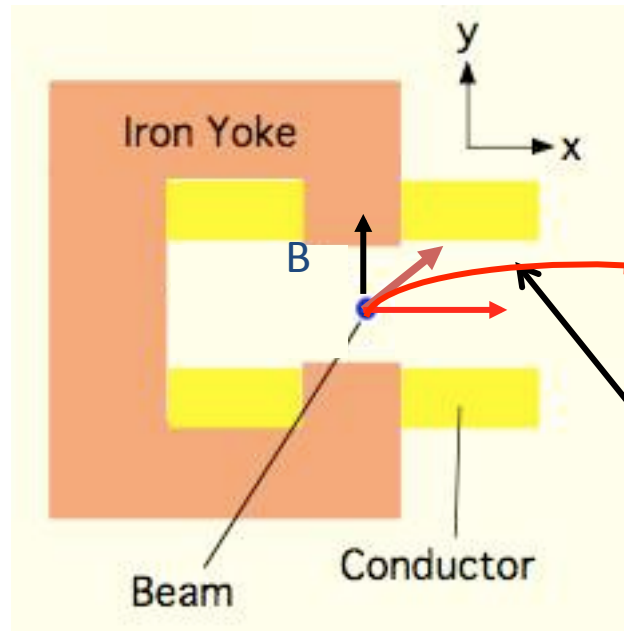
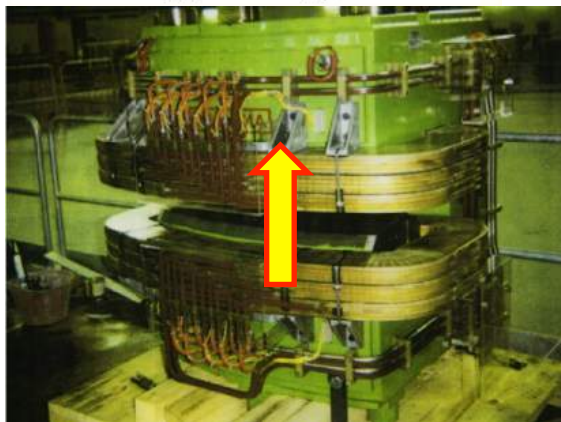
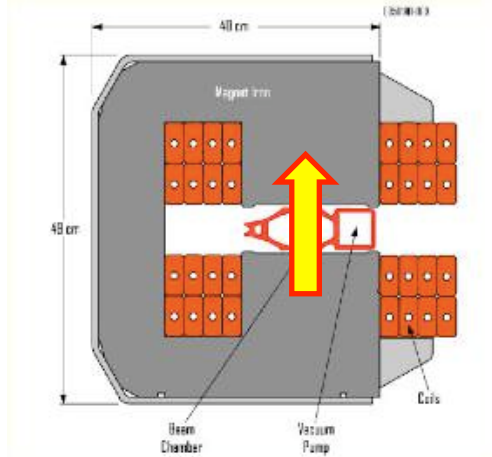
**E. O. Lawrence (1930)** had the idea to curve particles on a circular trajectory in order to pass many times in the same accelerating system gaining energy at each passage



Particles traveling in a **linear accelerator** pass through the accelerating structure only **once**, while in a **circular accelerator** they pass into the same cavity **several times** and at each turn, bunches gain energy thanks to the RF accelerating electric field.

# DIPOLES: DEFLECTION

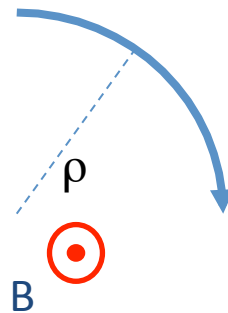
Dipoles allow to curve the trajectory of the particles. They can be made with **permanent magnets** or electromagnets (iron poles with **current coils**).



Circular trajectory

Radius of curvature

$$\rho [m] = \frac{p}{Bq} \cong \frac{E}{cqB}$$



For ultra-relativistic particles ( $v \cong c$ )

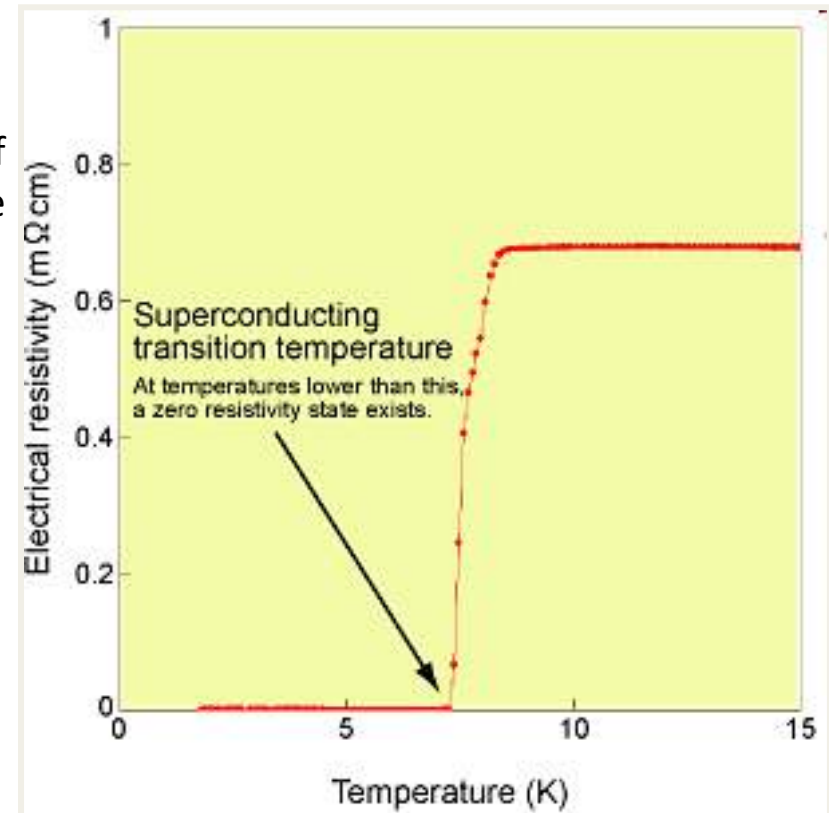
**Exercise:** calculate the magnetic field necessary to bend the DAFNE electron beam (510 MeV) to a circular trajectory of about 100 m.

# SUPERCONDUCTORS

Electromagnetic dipoles are used to obtain B field up to about 1-2 T.  
For larger field superconducting magnets are necessary

**Superconducting** materials below a certain temperature (of the order of few Kelvin) offer a **negligible resistance** to the passage of the current.

They can be used to build magnets with B field **up to 10 T**.

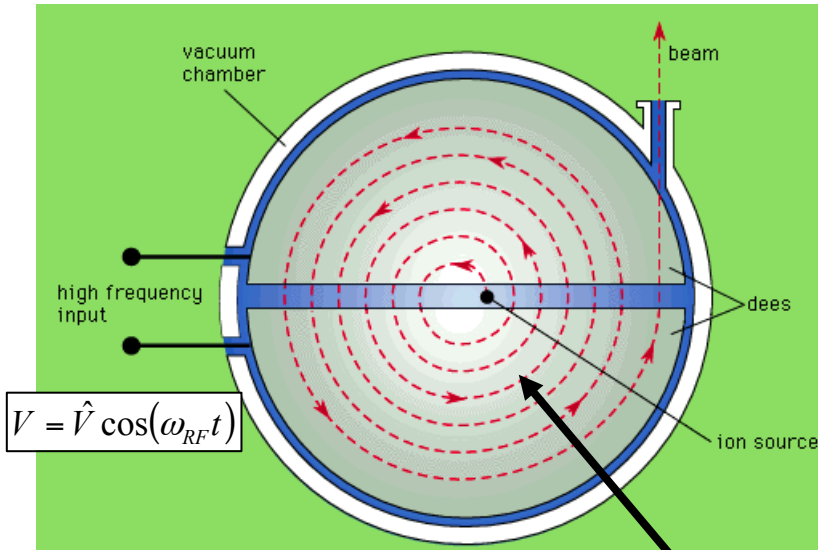


These temperatures are obtained by cooling the conductors with a refrigerator device that uses superfluid He: the **cryostat**.



# CYCLOTRONS (1/2)

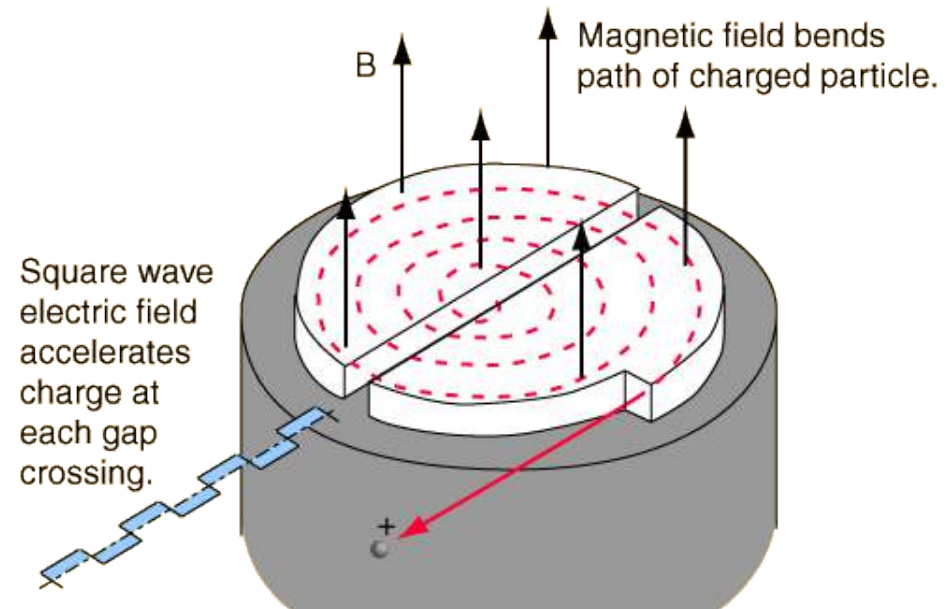
In **cyclotrons** acceleration is achieved by an **alternating electric field** between two or more electrodes immersed in a **constant dipolar magnetic field** (E.O.Lawrence-1930).



Spiral trajectory  $\rho = \frac{mv}{qB}$

**Energy gain** at each gap passage

$$\Delta E = q\hat{V} \cos \phi$$



The **synchronism** between the accelerating field and the particles is maintained if the relation is satisfied:

$$f_{RF} = hf_{rev} = h \frac{qB}{2\pi m}$$

Standard cyclotrons have a **constant  $f_{RF}$**  and, therefore, this synchronism is perfectly maintained only in the case of **non-relativistic particles** ( $m = m_0 = \text{constant}$ )

# CYCLOTRONS (2/2)



Since cyclotrons accelerate particles on a circular (spiral) path, it is possible to obtain **long distances in a small space**.

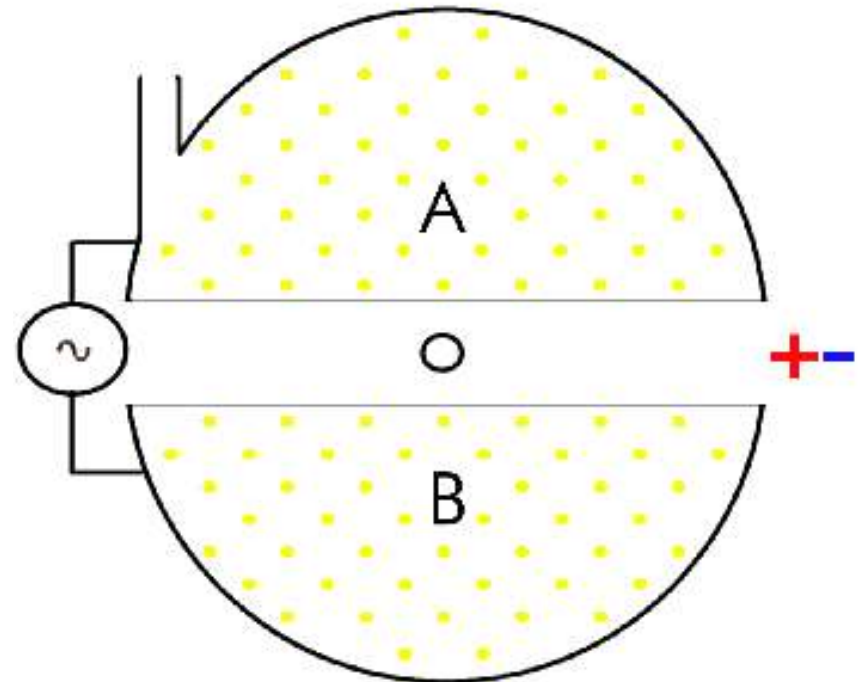
It can be fed with a **single and relatively cheap electronic system**.

The main problem is that, in order to obtain high energies, it is necessary to increase the diameter of the vacuum chamber and of the magnet

It finds many applications in the **first ion acceleration**.

The cyclotron was designed with the intention to **overcome the limitations of linear accelerator**.

At that time (1930) it was not possible to generate radiofrequency field at high frequency and high power, so to obtain high energies it was necessary to build long accelerators and, beyond a certain limit, too expensive.



<https://youtu.be/cutKuFxeXmQ>

# MAGNETIC QUADRUPOLE

Quadrupoles are used to **focalize the beam** in the transverse plane. It is a **4 poles magnet**:

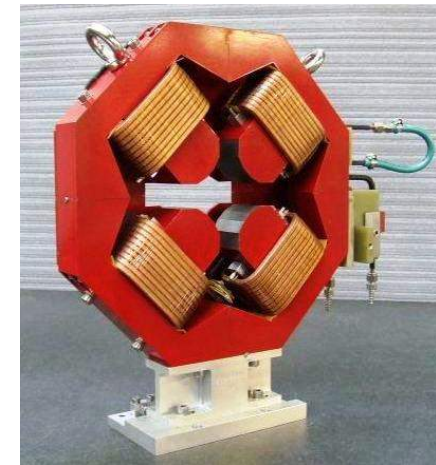
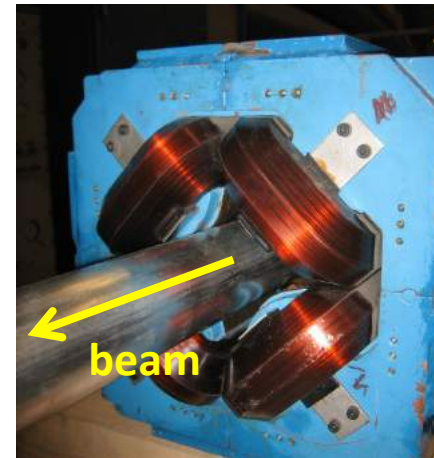
⇒ **B=0** in the center of the quadrupole

⇒ The **B intensity increases linearly** with the off-axis displacement.

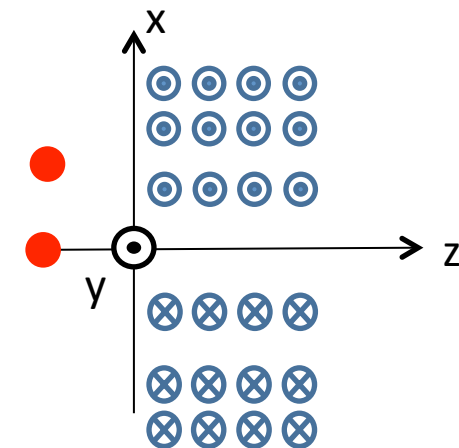
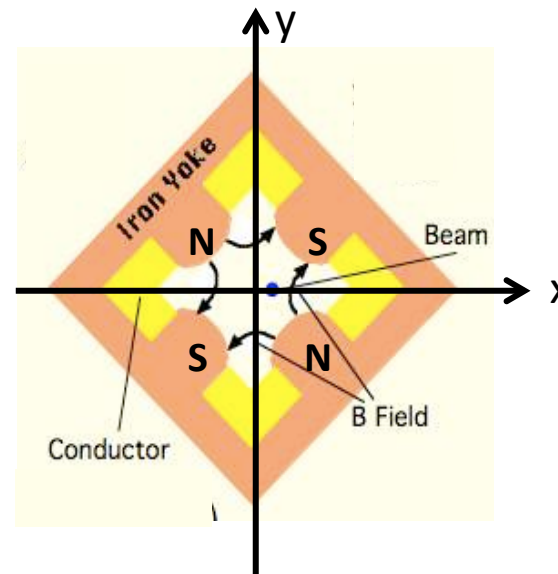
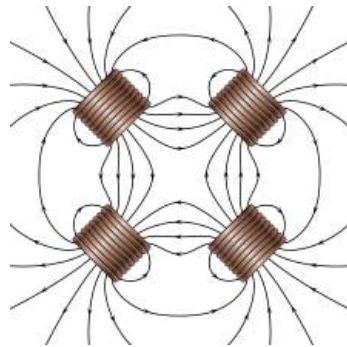
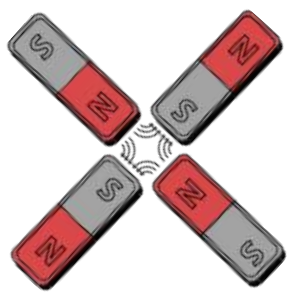
⇒ If the quadrupole is **focusing in one plane** is **defocusing in the other plane**

$$\begin{cases} B_x = G \cdot y \\ B_y = G \cdot x \end{cases} \Rightarrow \begin{cases} F_y = qvG \cdot y \\ F_x = -qvG \cdot x \end{cases}$$

$$G = \text{quadrupole gradient} \left[ \frac{T}{m} \right]$$



Electromagnetic quadrupoles  $G < 50-100 \text{ T/m}$

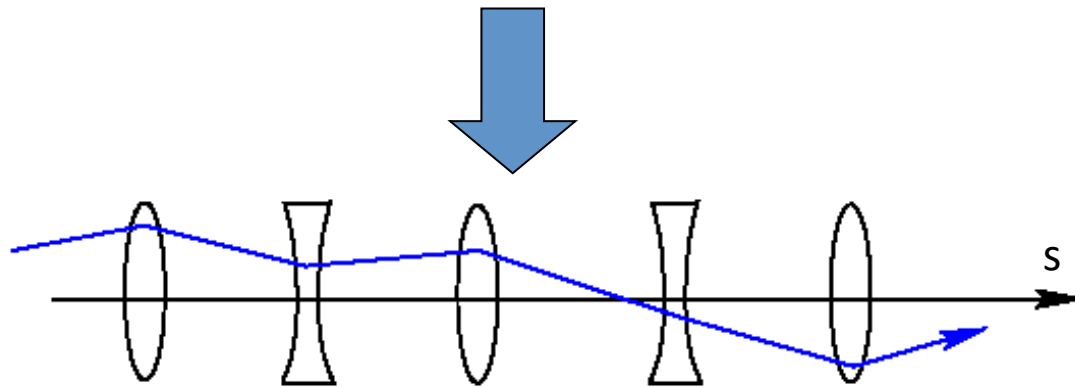


# TRANSVERSE FOCUSING: $\beta$ -FUNCTION

A quadrupole focus in one plane and de-focus in the other plane.

To obtain the **overall focus** of a particle beam along a transport channel or in a circular accelerator it is necessary to use a **sequence of quadrupoles with the alternating sign**.

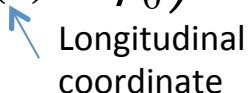
This configuration is able to guarantee **stable trajectories**.

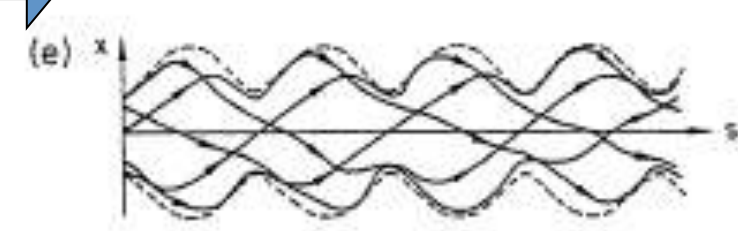
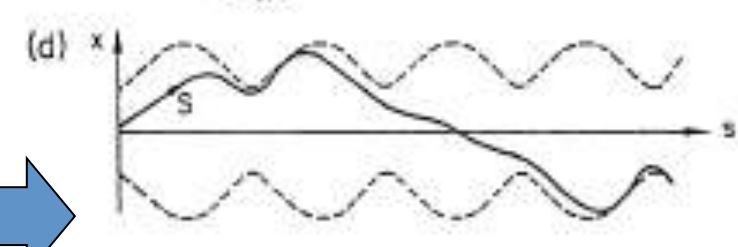
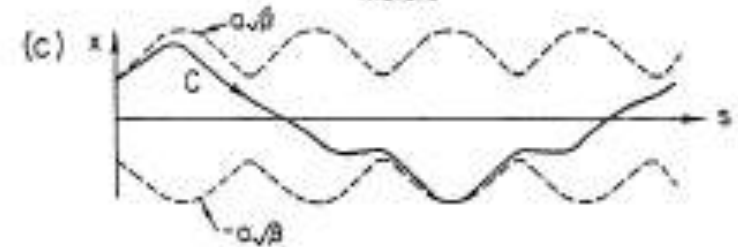
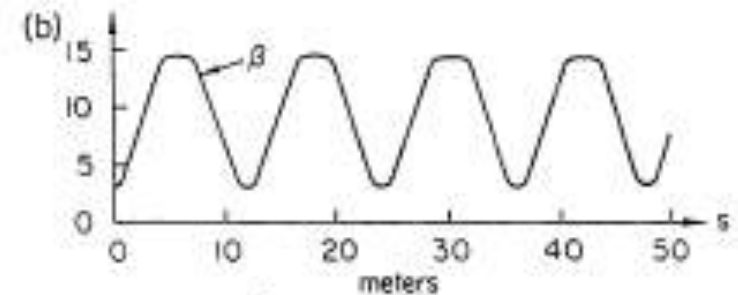


The transverse trajectory described by each particle is a **pseudo-sinusoid**.

The **envelope** of all trajectories is called the  **$\beta$ -function**.

$$x(s) = \sqrt{\beta(s)\epsilon(s)} \cos(\varphi(s) + \varphi_0)$$

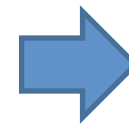
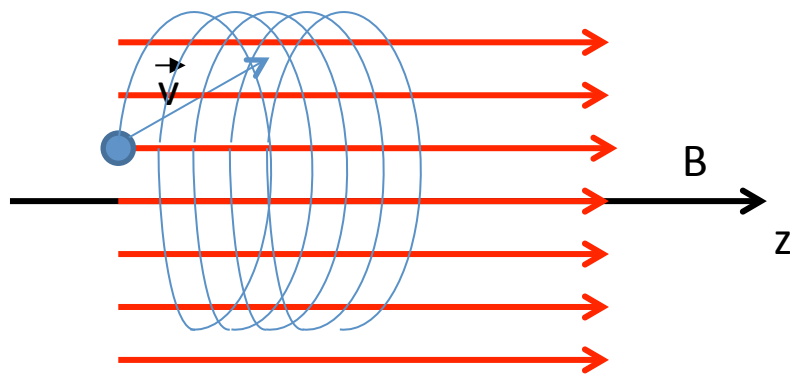
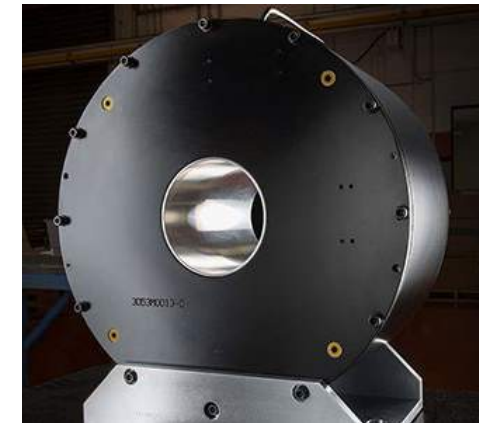
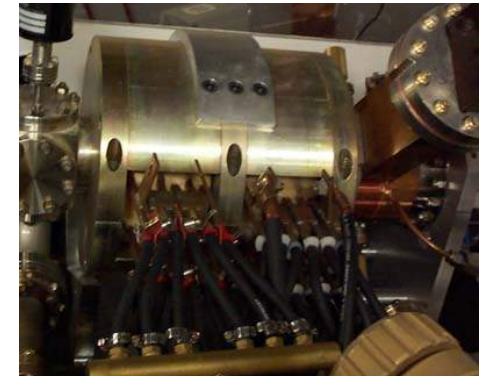
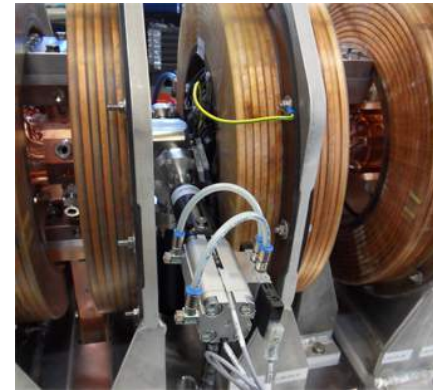
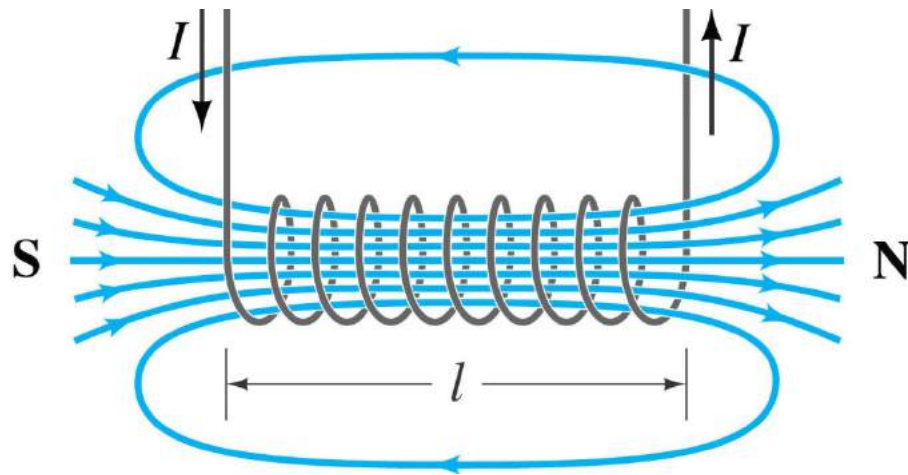

 Longitudinal coordinate





# SOLENOID

Also solenoids can be used for focalization of beams (in particular electron beams).



Useful for beam emittance compensation at low energies

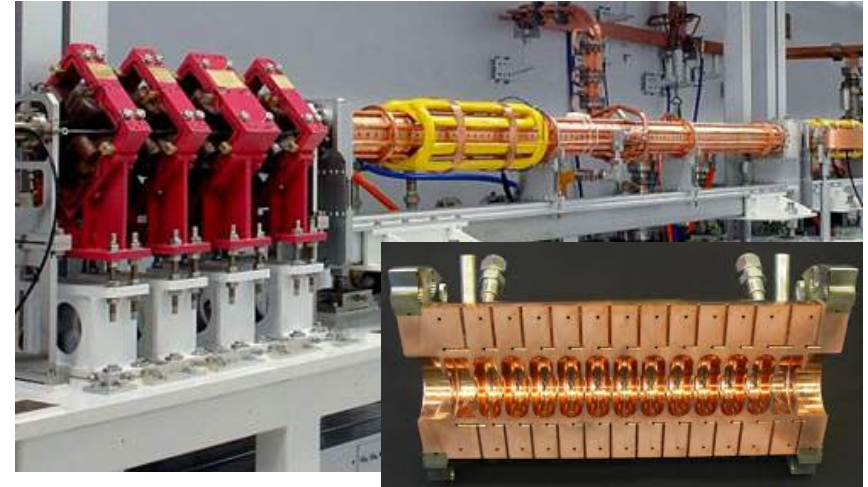
Particles that enter into a solenoidal field with a transverse component of the velocity (divergence) start to **spiralize describing circular trajectories** .

# LINAC AND SYNCHROTRON

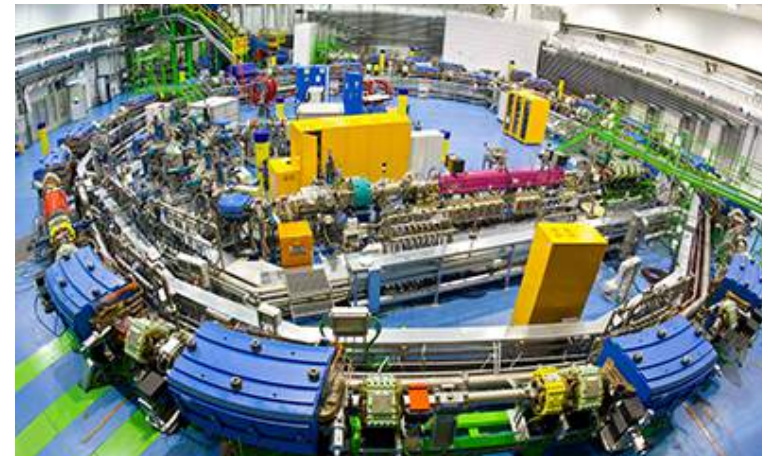
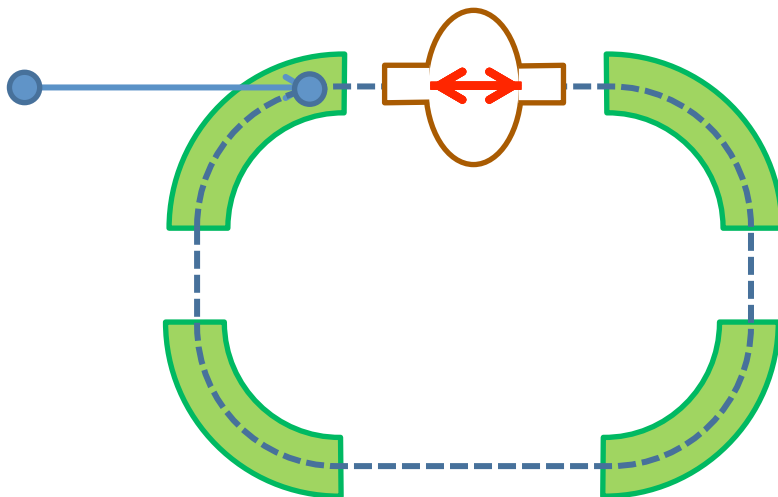
A typical LINAC is, in conclusion, an **alternating sequence** of accelerating sections, **quadrupoles**, **diagnostic elements** (that allow the measurement of the beam position, profile, charge, etc...) and **pumping systems**. Vacuum in a particle accelerator is typically on the order of  $10^{-8}$ - $10^{-10}$  mbar. If such pressures were not reached, the particles would be lost due to impacts with the gas molecules. They can be of few m up to km...



m  km



By **dipoles** the beam can also be circulated in a ring. A single **accelerating cavity** accelerate the particles at each passage. These machines are called **synchrotrons**.

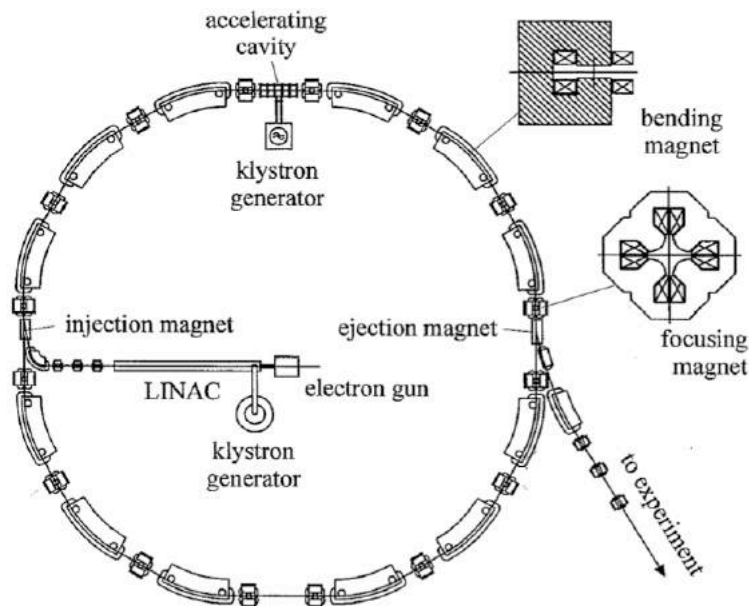




# CIRCULAR ACCELERATORS: SYNCHROTRON

**Synchrotron is a circular particle accelerator.** In synchrotrons, the particles describe closed orbits thanks to the use of dipoles. The accelerating electric field is synchronized with the particle beam so that at each subsequent passage in cavities they increase their energy.

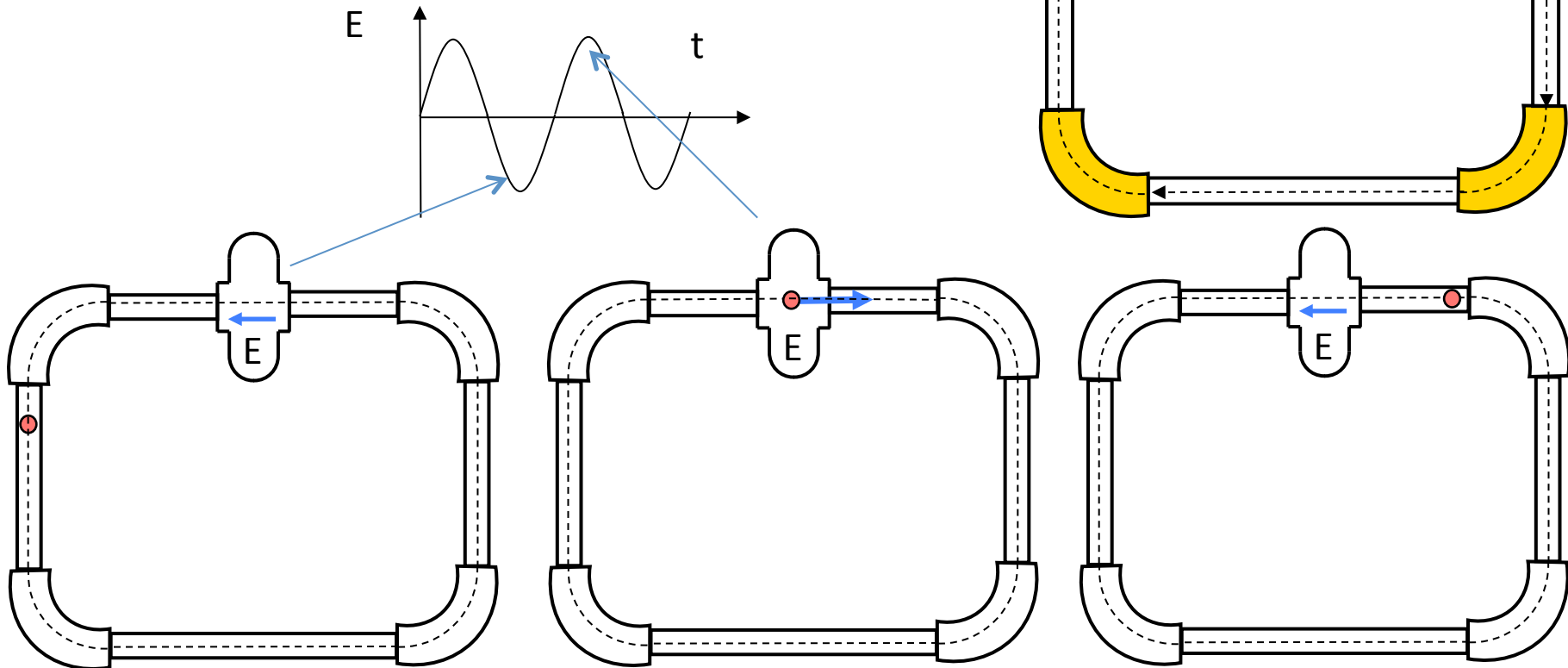
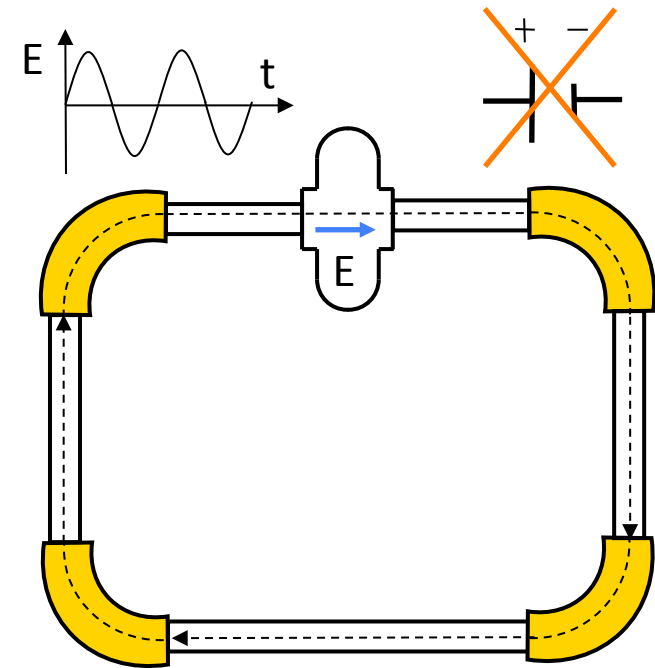
**DIPOLES:** bend the particles and determine the reference circular trajectory  
**QUADRUPOLES:** keep the oscillations of all the particles around the reference trajectory and inside the vacuum chamber  
**SESTUPOLES:** correct the chromatic effect of the quadrupoles  
**CAVITY RF:** accelerates the beam  
**VACUUM CHAMBER AND PUMPING**  
**DIAGNOSTICS**



# ACCELERATION IN A SYNCHROTRON

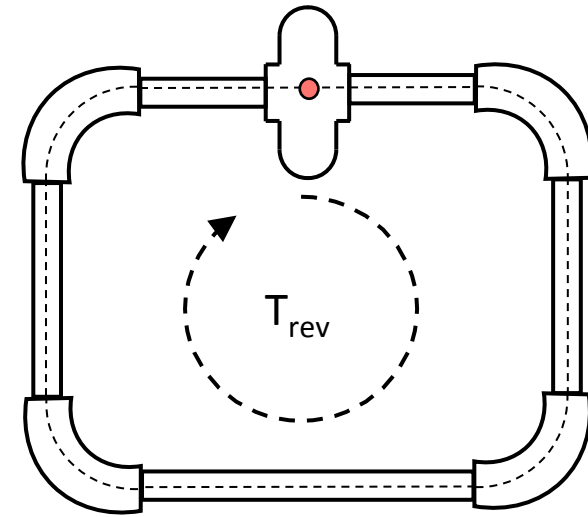
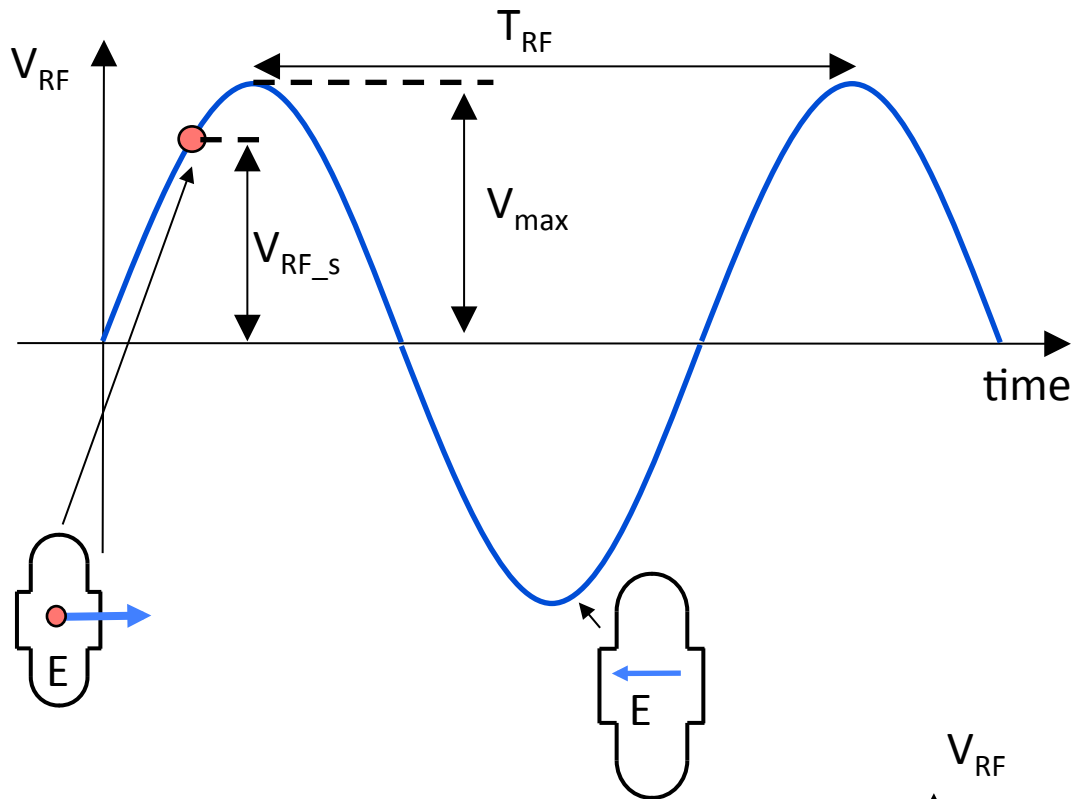
The **electric field** in the cavity accelerates particles.

It can not be electrostatic but has to **oscillate** otherwise, in one complete turn, a particle would gain energy in the cavity and lose it in the remaining part of the accelerator-principle (electrostatic field is conservative)





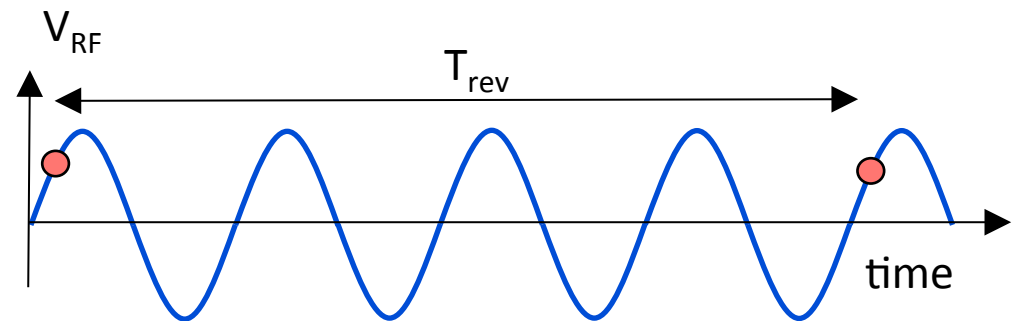
# REVOLUTION PERIOD AND HARMONIC NUMBER



The time necessary to complete one turn is called **revolution period** ( $T_{rev}$ )

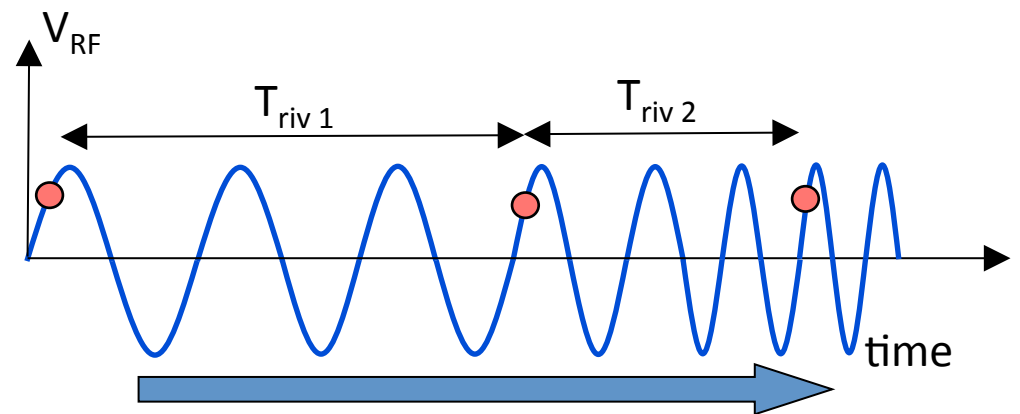
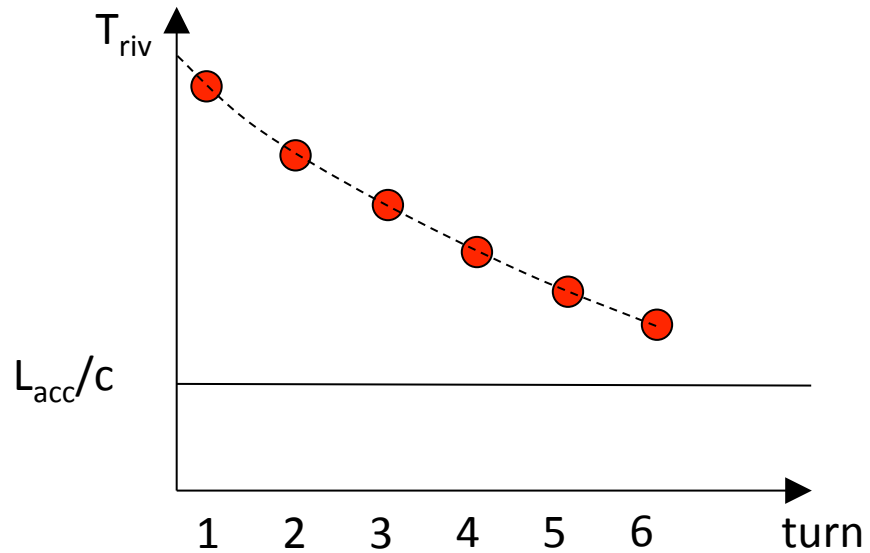
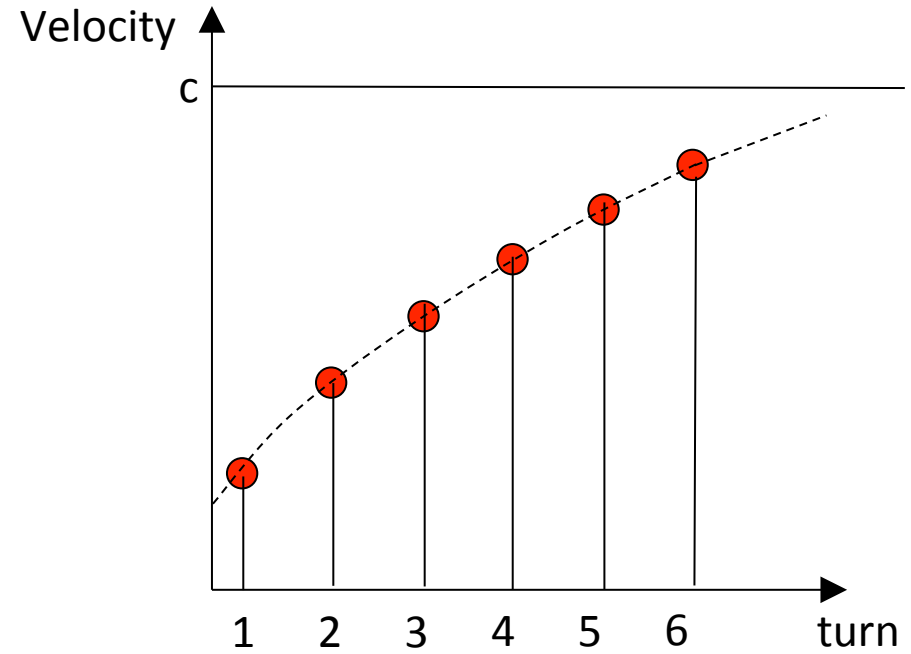
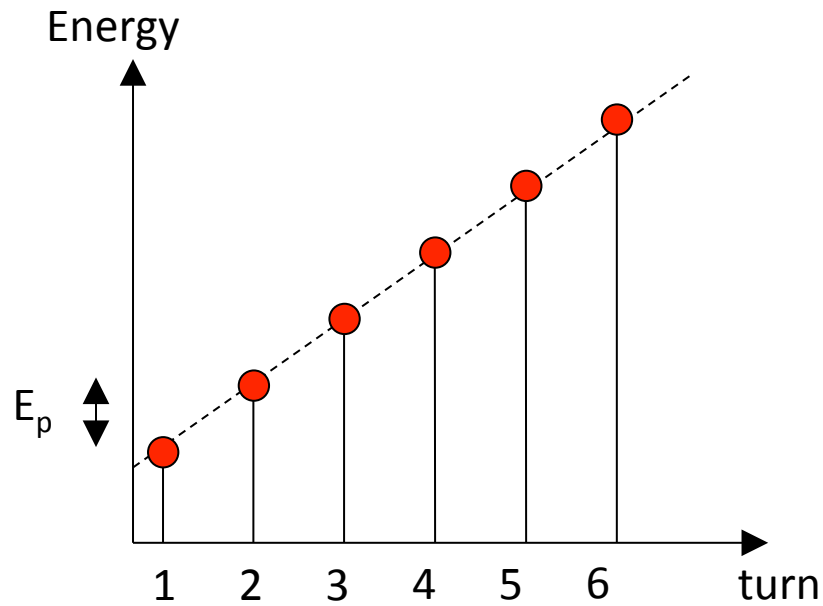
To have "stable" acceleration, the **revolution period** ( $T_{rev}$ ) must be an integer multiple ( $h$ ) of the radiofrequency period ( $T_{RF}$ ).

$h$  is called the **harmonic number** and is equal to the number of "bunches" of particles that can be simultaneously accelerated in the synchrotron.



$$T_{rev} = hT_{RF}$$

# ACCELERATION, ENERGY AND SPEED



During acceleration the RF frequency has to increase to maintain synchronism.

# LORENTZ FORCE: ACCELERATION AND FOCUSING

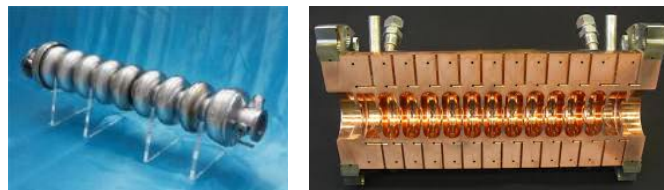
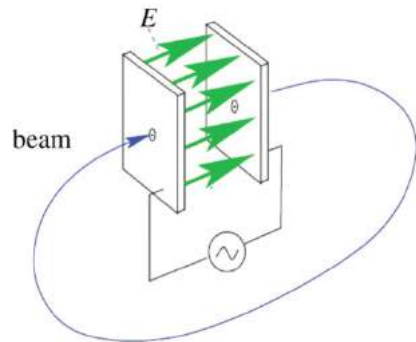
Particles are accelerated through electric field and are bended and focalized through magnetic field. The basic equation that describe the acceleration/bending /focusing processes is the **Lorentz Force**.

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$$\frac{d\vec{p}}{dt} = q \left( \vec{E} + \vec{v} \times \vec{B} \right)$$

## ACCELERATION

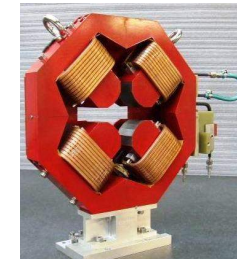
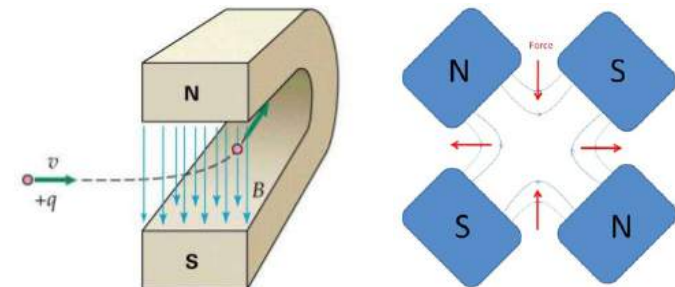
To accelerate, we need a force in the direction of motion



Longitudinal Dynamics

## BENDING AND FOCUSING

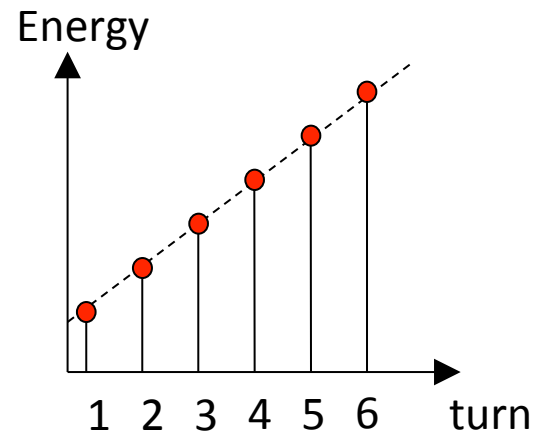
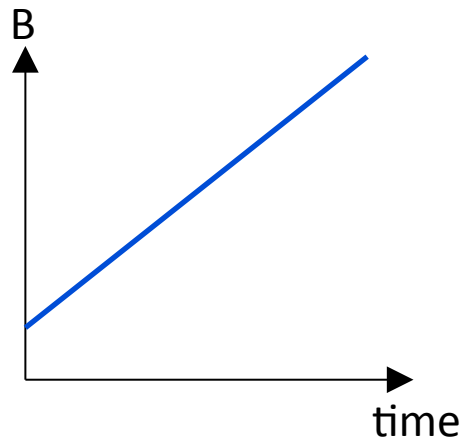
2<sup>nd</sup> term always perpendicular to motion => no energy gain



Transverse Dynamics

# MAGNETIC FIELD

The energy increase at each turn has to correspond to an **increase in the intensity of the magnetic field of the dipoles (B)** in order to keep the particles always on the same orbit.



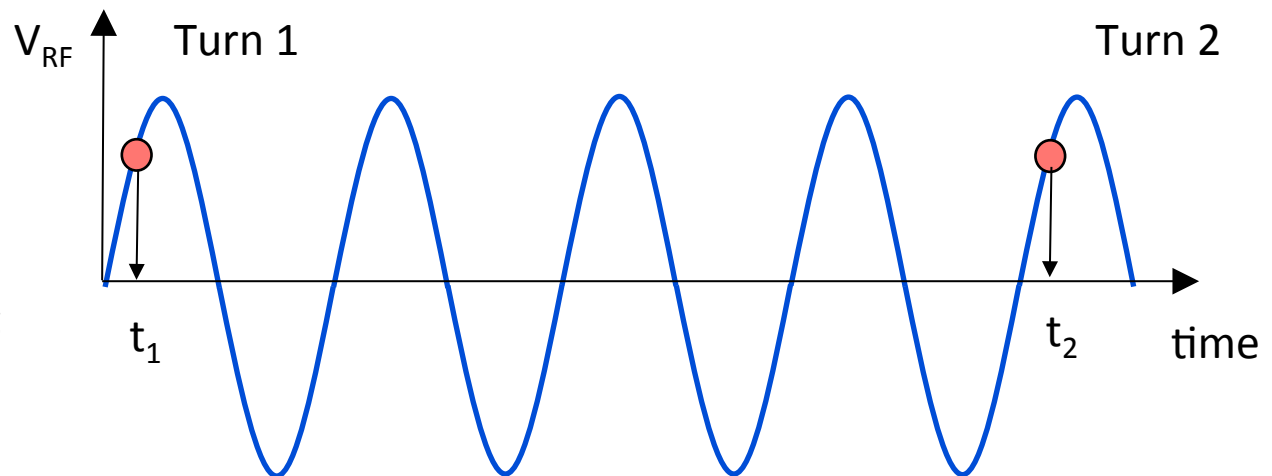
$$\rho \propto \frac{E_{particle}}{B}$$

Radius of curvature

# SYNCHRONOUS PARTICLE

The **synchronous particle** is the particle that at each turn:

- 1) has the **nominal energy** that allows it to always describe the same reference orbit;
- 2) it always enters the cavity with the same **phase with respect to the accelerating voltage** (synchronous phase) and always sees the same accelerating voltage, gaining the same energy.





# NON SYNCHRONOUS PARTICLE

The **motion of a non-synchronous particle** ("near" in position and energy to the synchronous one) can be described with the two following variables:

Arrival time of the non synchronous particle in the cavity

Arrival time of the synchronous particle in the cavity

$$t - t_s = \tau$$

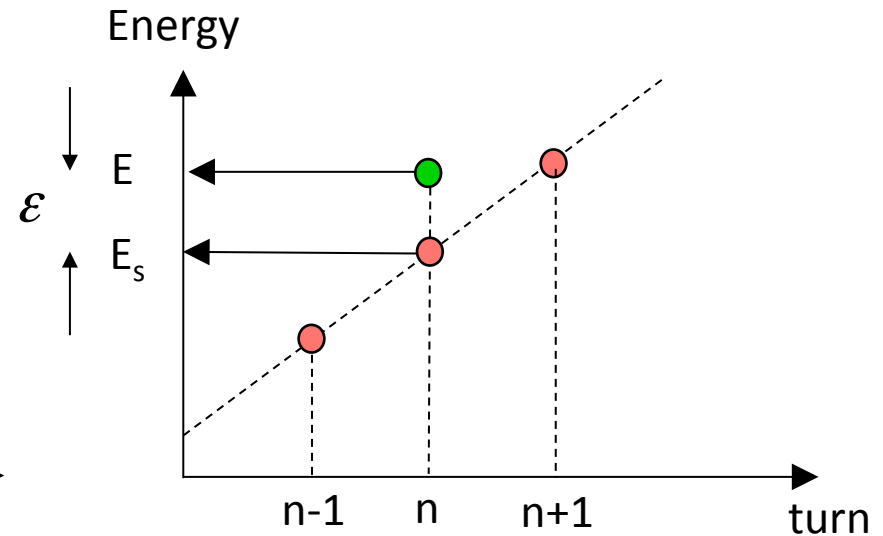
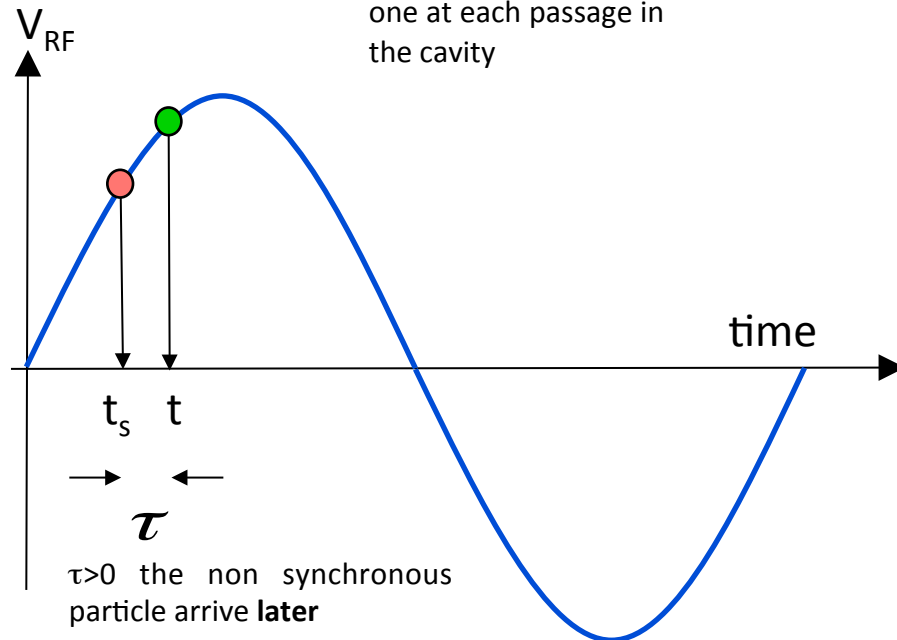
Time distance between the non synchronous particle and the synchronous one at each passage in the cavity

Energy of the non synchronous particle at passage in the cavity

Energy of the synchronous particle at passage in the cavity

Difference between the two energies

$$E - E_s = \varepsilon$$



# SYNCHROTRON OSCILLATIONS

We can consider a particle entering the cavity **after synchronous one**.

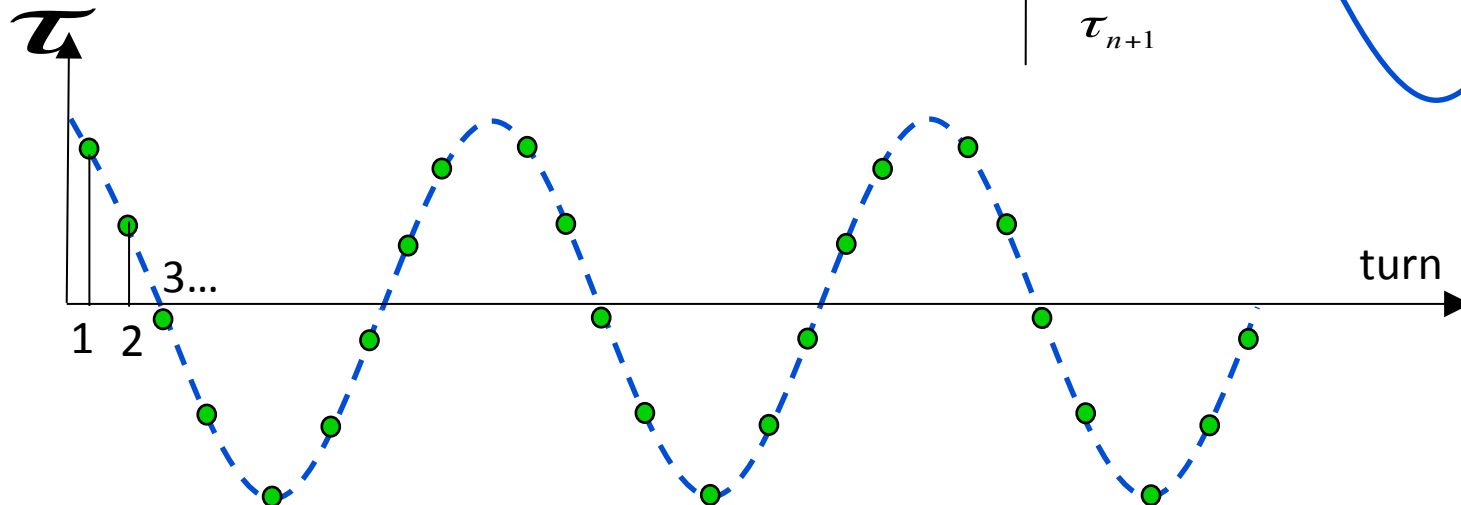
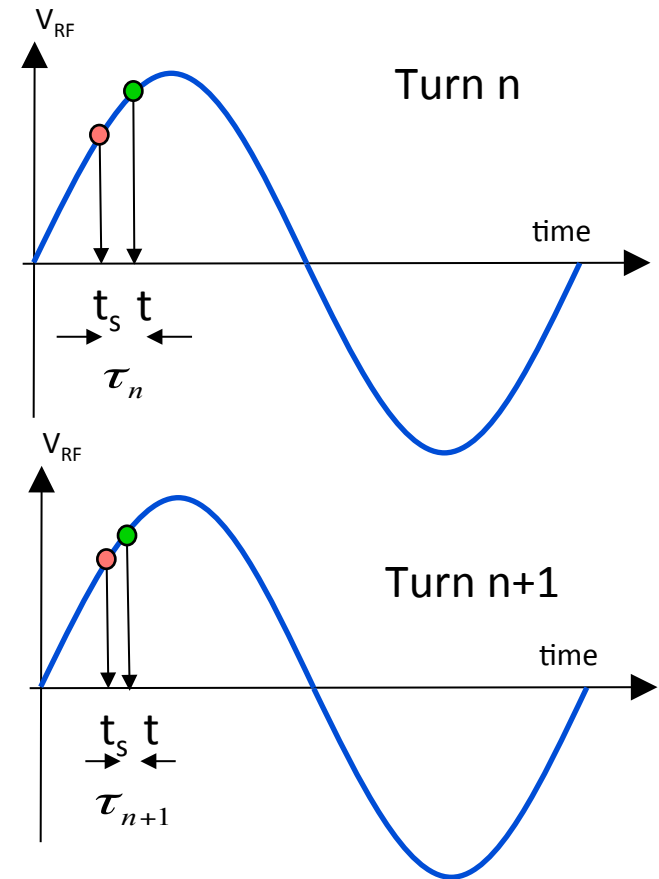
The **accelerating field** seen by the particle is **larger** than that seen by the synchronous particle.

This increased acceleration corresponds to an **increase in speed** and, therefore, at the next turn it will have recovered part of its "disadvantage" and will be closer.

On the other hand, if a particle arrives before the synchronous particle in the cavity it sees a voltage smaller and will gain less energy.

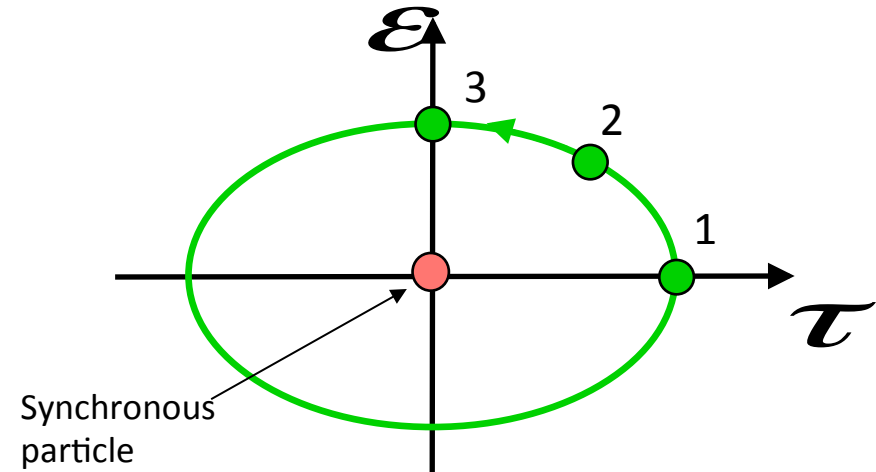
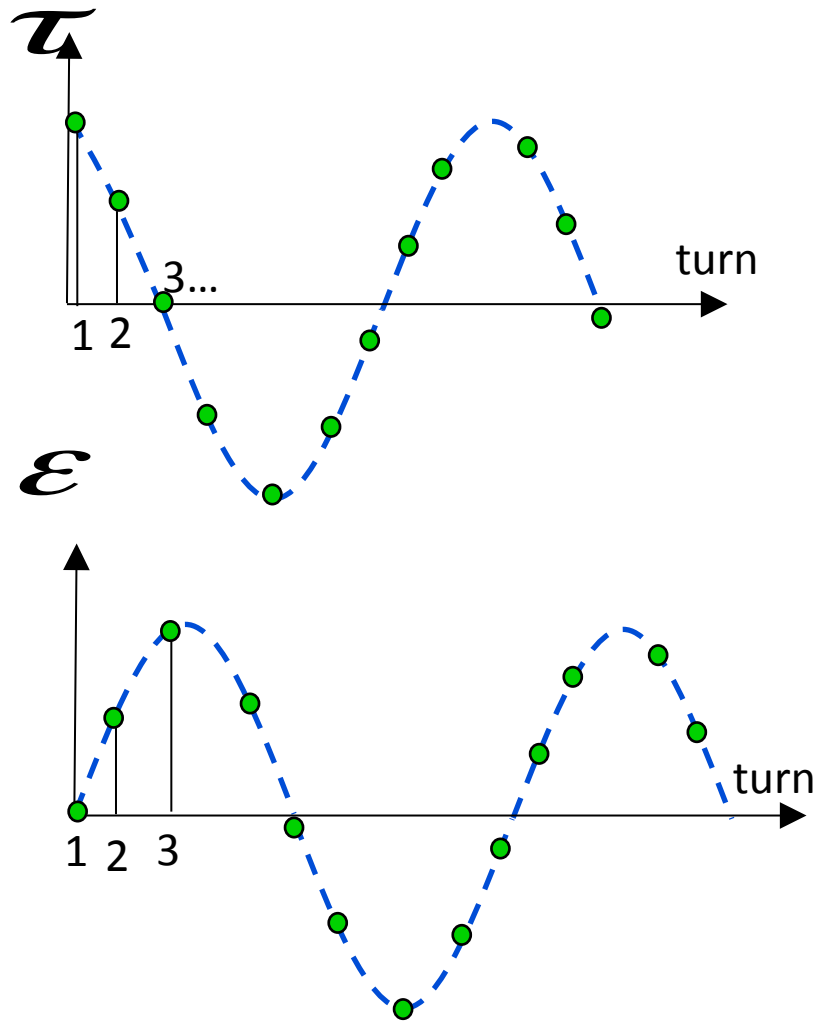
In other words **all particles oscillate around the synchronous one**.

These oscillations are called **synchrotron oscillations** and the corresponding frequency is called synchrotron frequency.



# SYNCHROTRON OSCILLATIONS IN THE PHASE SPACE

It is clear that the **accelerating voltage** acts like a restoring force similar to that of a **spring**



In the plane  $(\tau, \epsilon)$  referred as “**longitudinal phase space**”, a non-synchronous particle describes an ellipse with a frequency equal to the synchrotron frequency. Typical synchrotron frequencies are of the order of tens of kHz.

$$f_s = f_{RF} \sqrt{\frac{\alpha_c V_{RF\_s}}{2\pi h E_s (eV)}}$$

$f_{RF}$  RF frequency

$\alpha_c$  momentum compaction  $\alpha_c = \frac{(L_E - L_{E_s})/L_{E_s}}{(E - E_s)/E_s}$

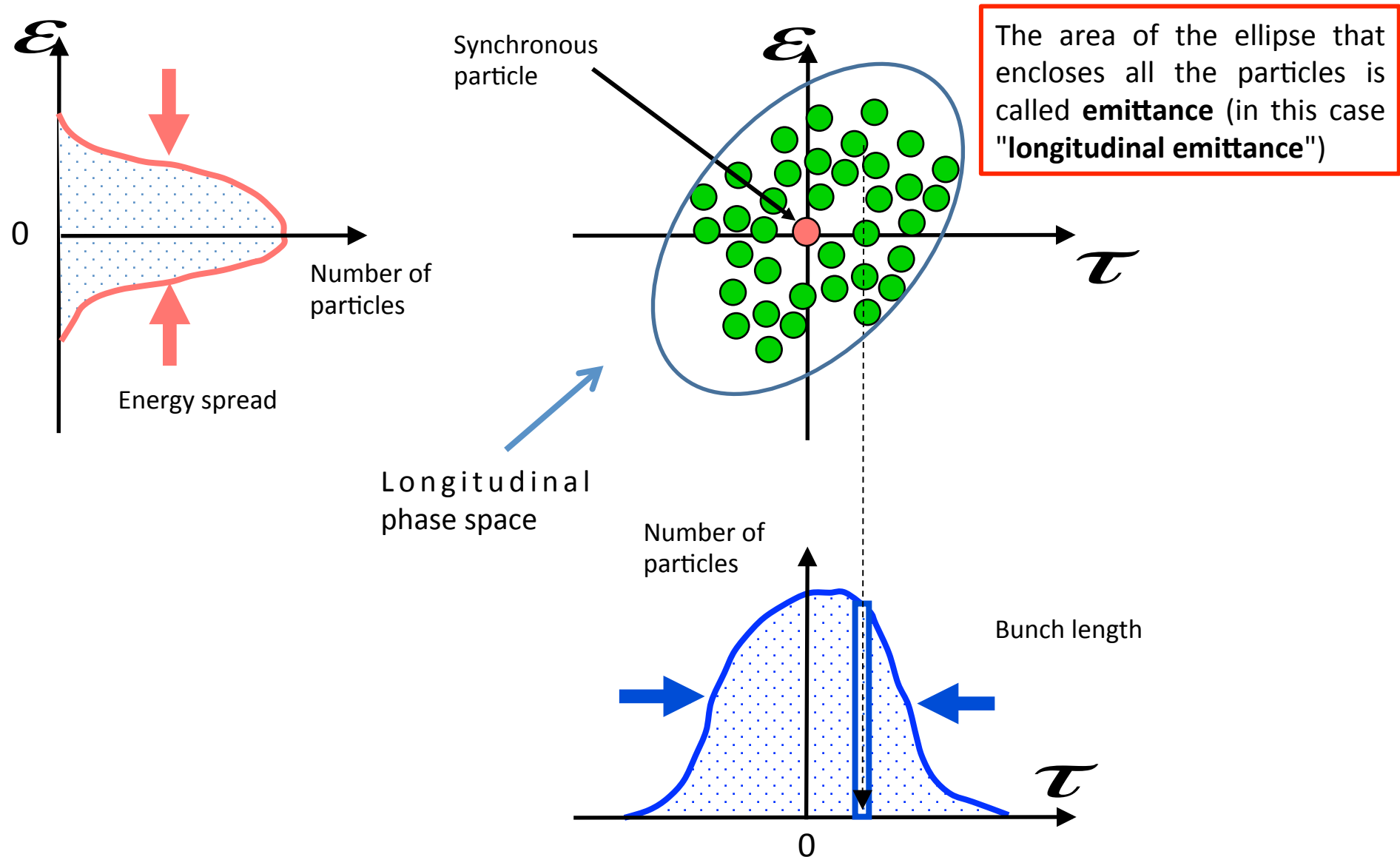
$V_{RF\_s}$  accelerating voltage

$h$  harmonic number

$E_s$  particle energy

# BUNCH LENGTH, ENERGY SPREAD AND EMITTANCE

The **N particles of the bunch** are distributed around the synchronous particle and oscillate around it describing ellipses.

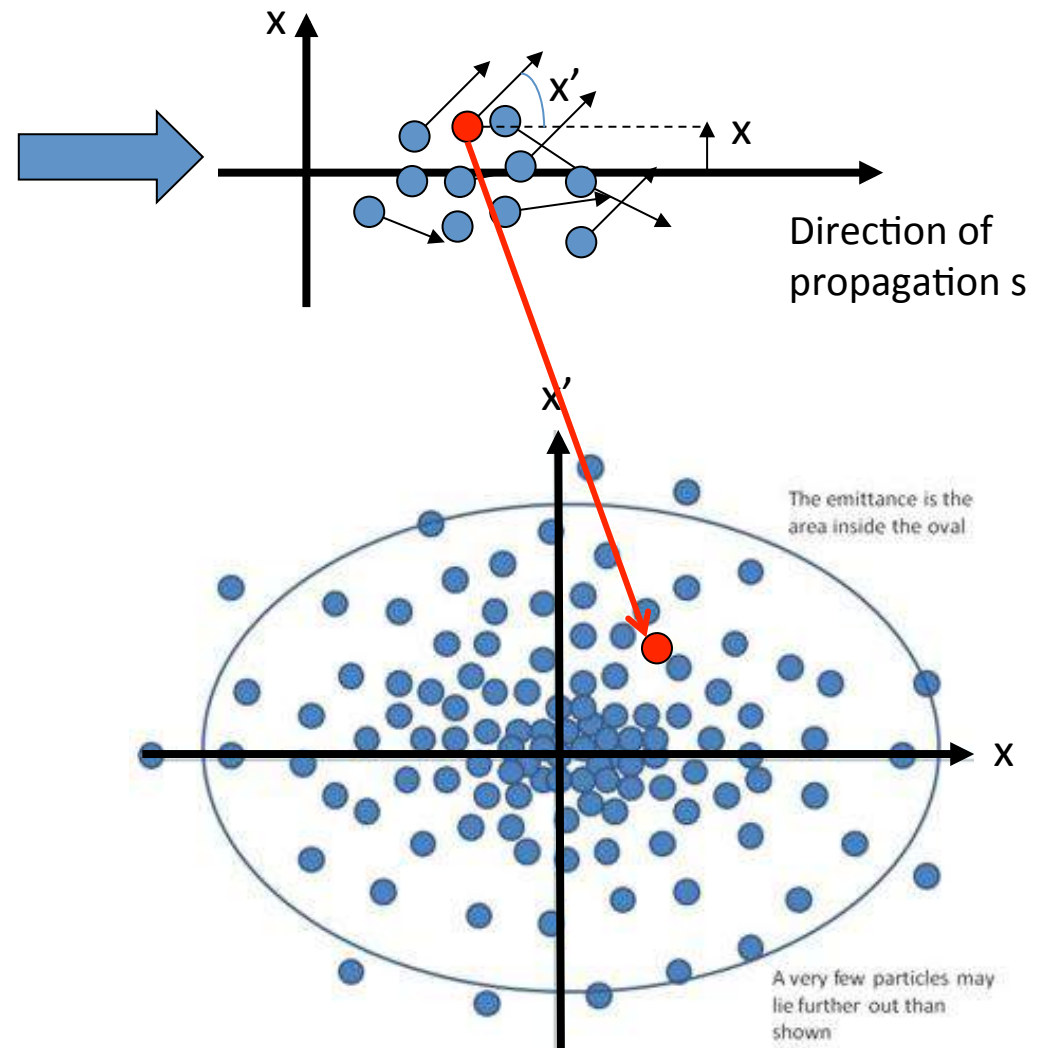
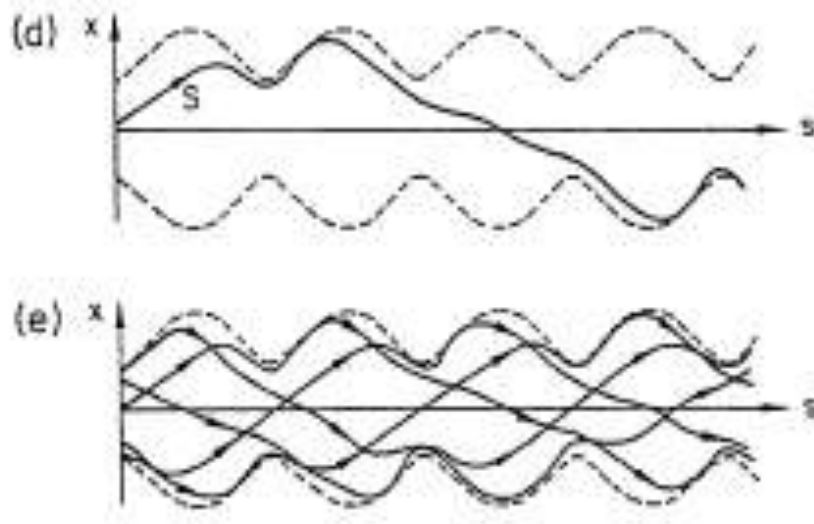




# TRANSVERSE PHASE SPACE AND EMITTANCE

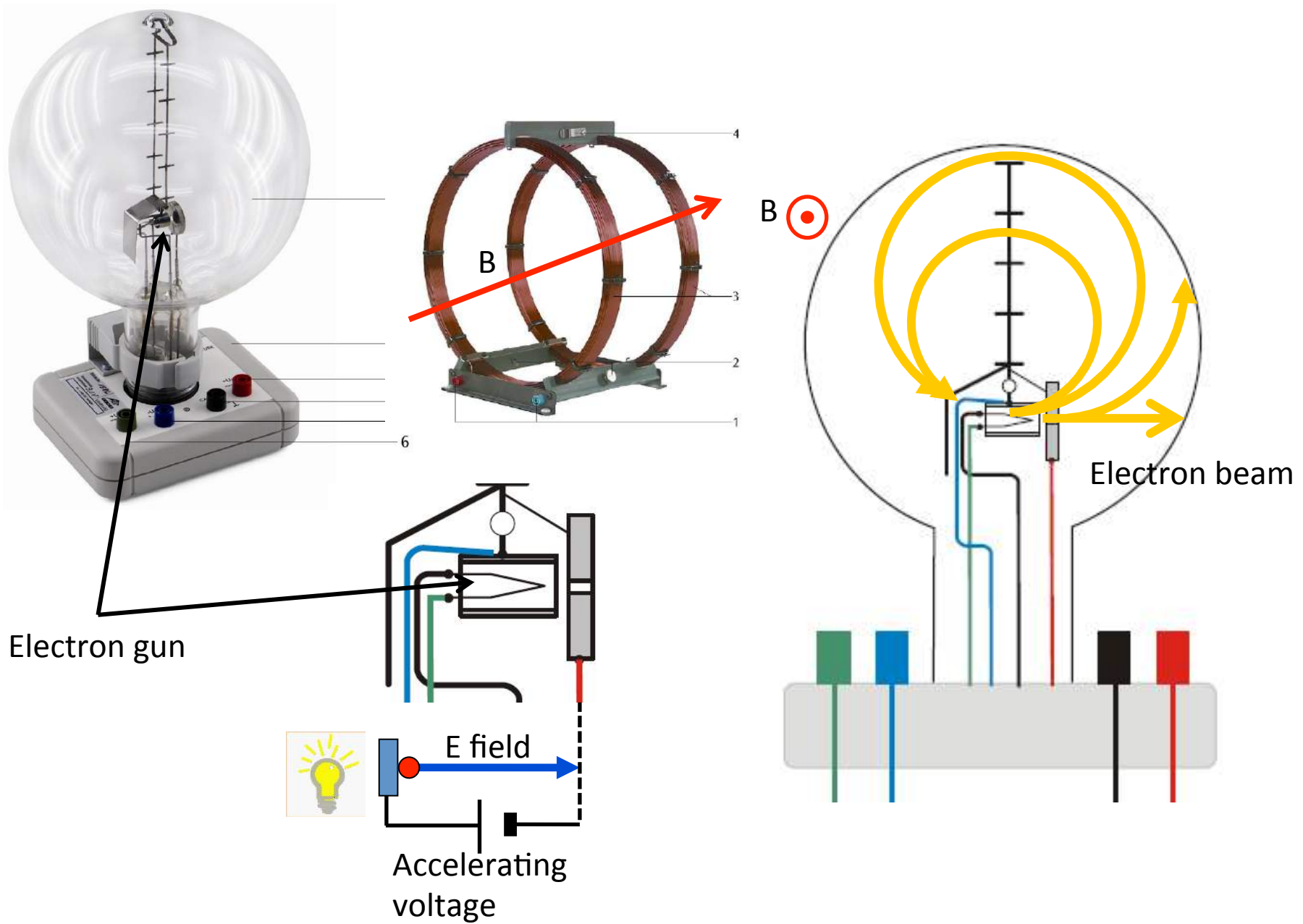
Similarly to what we have in the longitudinal plane also in the transverse plane the particles perform oscillations (called **betatron oscillations**) due to the force provided by quadrupoles.

At a certain point in the machine each particle has a given transverse position and angle with respect to the direction of propagation



The area in the phase space occupied by the bunch is called **transverse emittance**

# EXPERIMENT WITH e- BEAM



# EXERCISE: CALCULATIONS

$$\Delta V = 300V$$

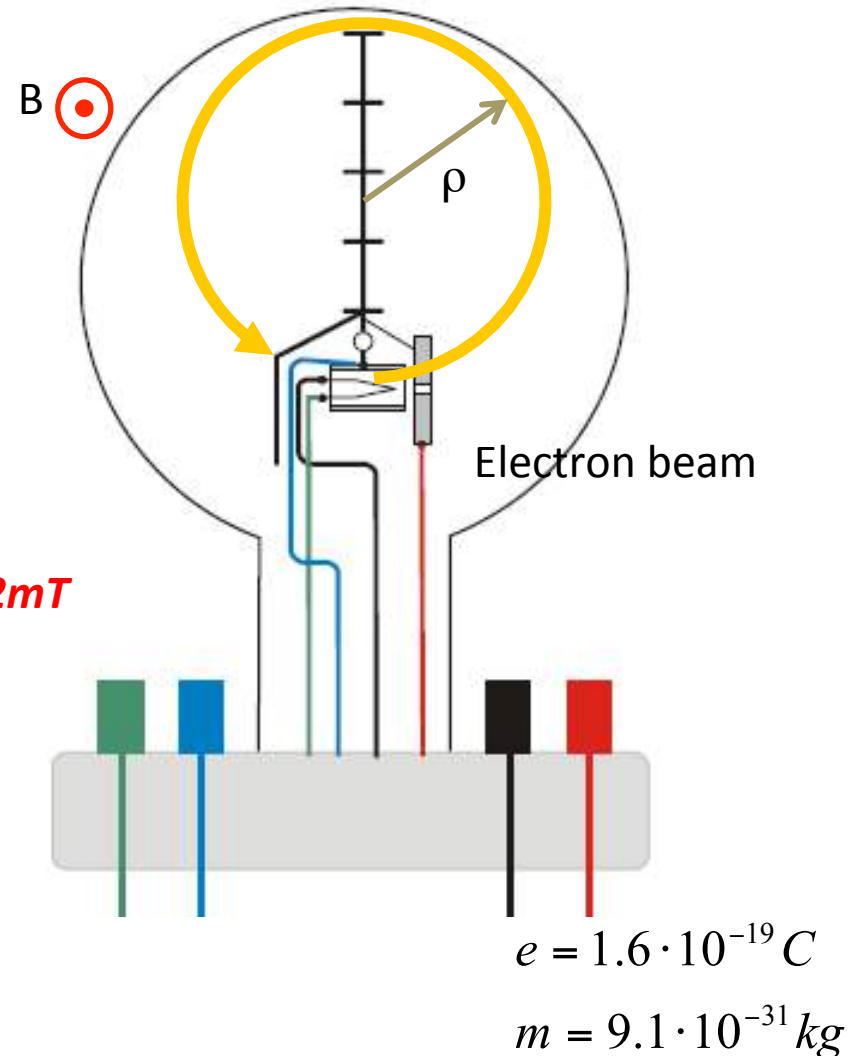
**Acceleration: calculate the particle velocity**

$$e\Delta V = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2\frac{e}{m}\Delta V} \Rightarrow v = 10.27 \cdot 10^6 \text{ m/s}$$

$$\beta = 0.034$$

**Deflection: calculate the radius of curvature with  $B=2\text{mT}$**

$$B = 2 \cdot 10^{-3} \text{ T} \Rightarrow \rho = \frac{mv}{Be} \cong 3\text{cm}$$



# SUMMARY: HOW WE ACCELERATE AND DEFLECT PARTICLES

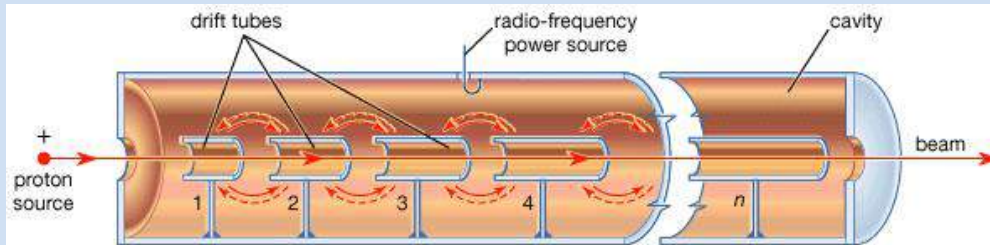
$\vec{p}$  = momentum

$\vec{v}$  = velocity

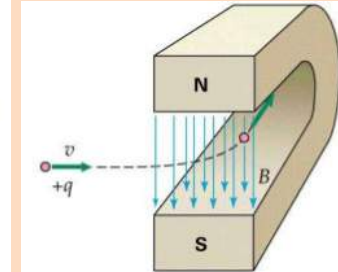
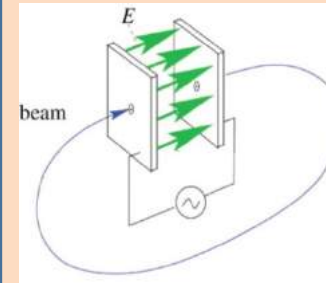
$q$  = charge

$$\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

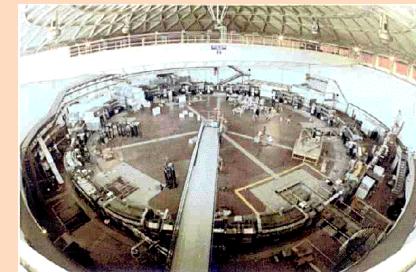
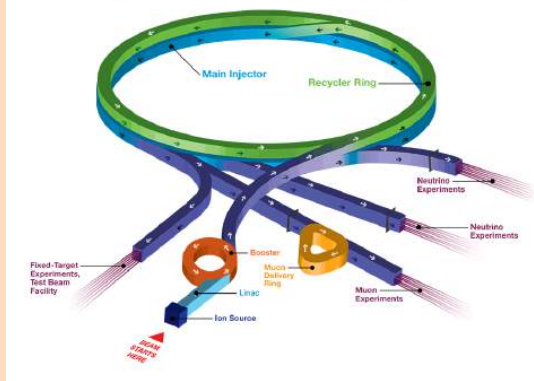
## LINEAR



## CIRCULAR

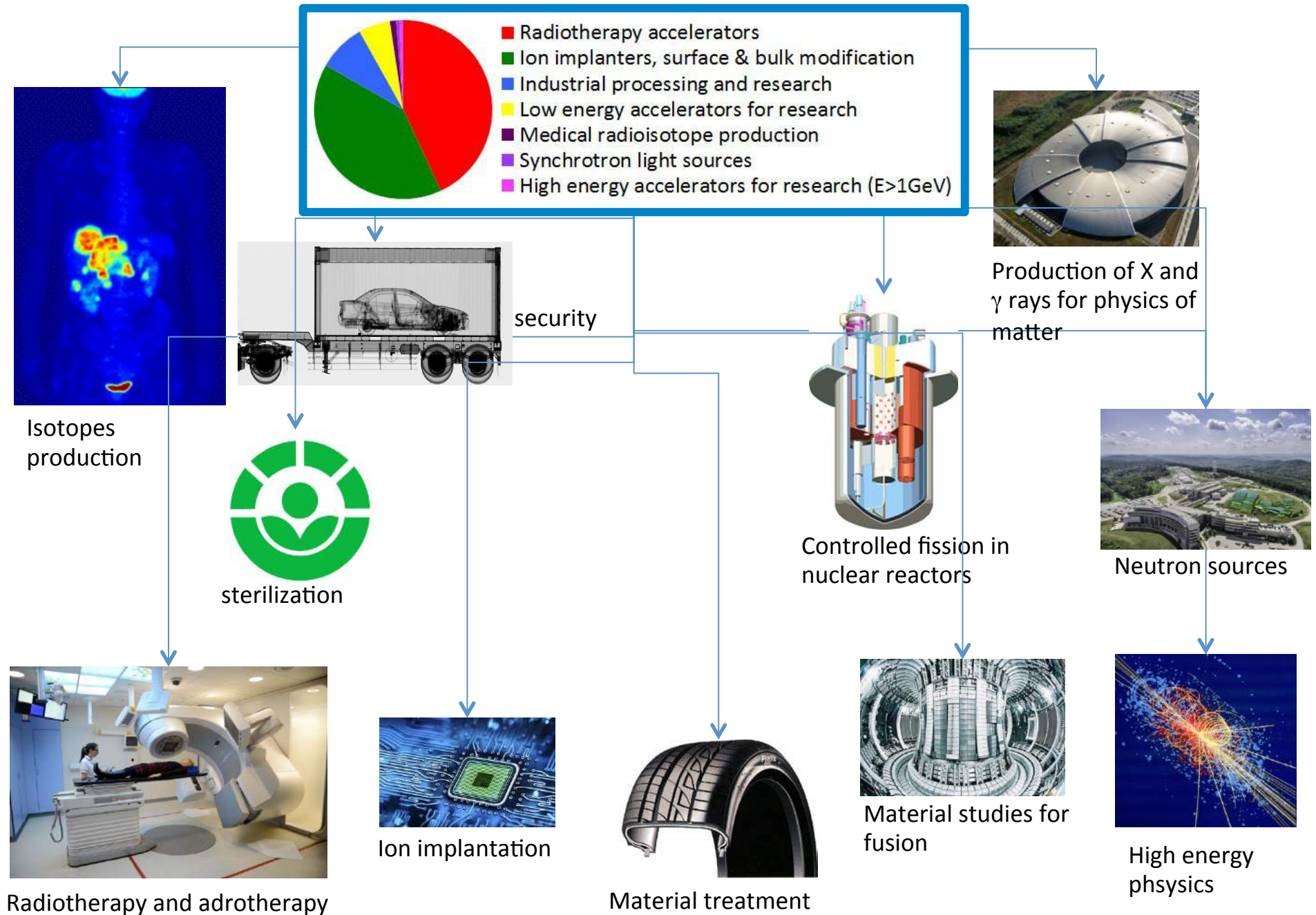


### Fermilab Accelerator Complex



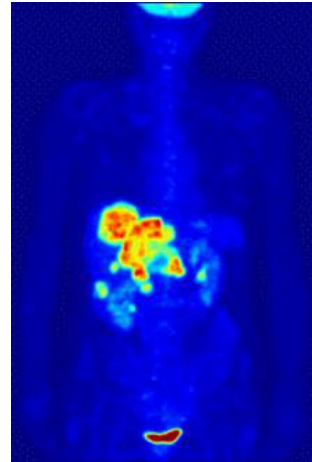
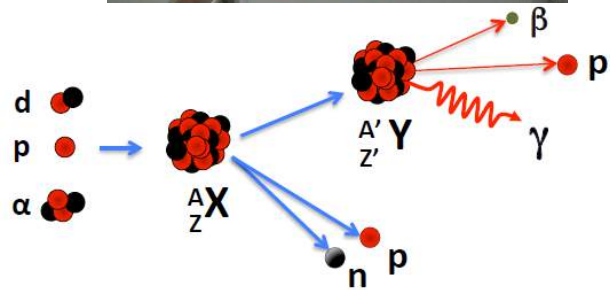


# WHAT CAN PARTICLE ACCELERATORS DO?

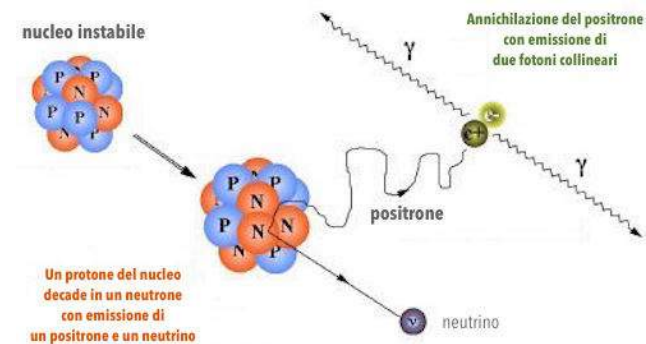
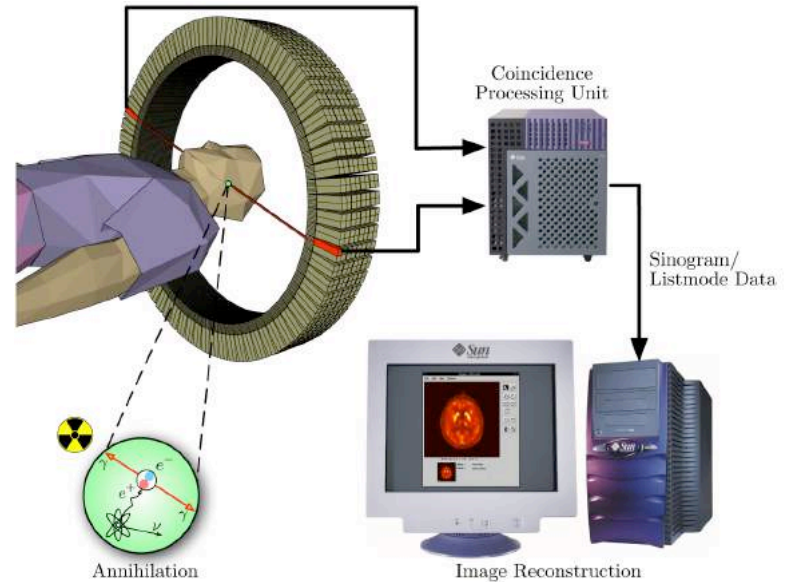


# MEDICAL APPLICATIONS: RADIOISOTOPE PRODUCTION

**Production of radioisotopes:** protons of 7-100 MeV accelerated with **cyclotrons or linacs** (50 type of isotopes, used for diagnostics and treatment are produced with accelerators)



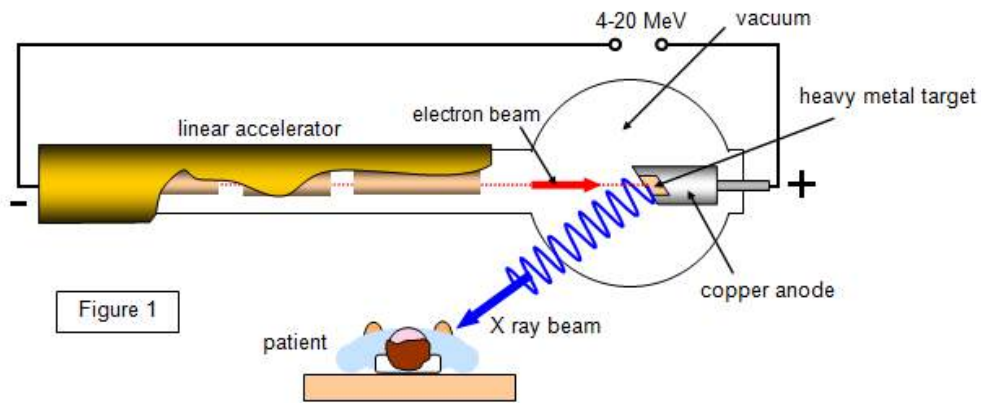
## PET: Positron Emission Tomography





# MEDICAL APPLICATIONS: RADIOTHERAPY

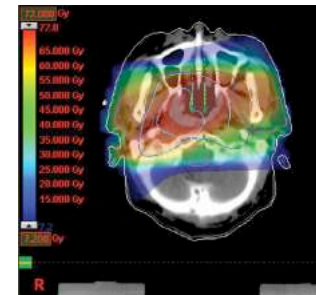
One irradiate tumors with X-rays or electrons



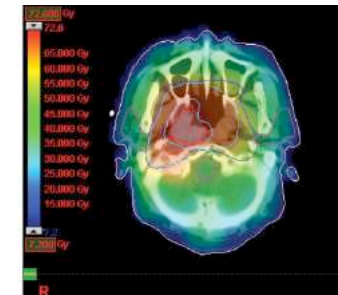
Metallic sheet for X ray production

LINAC

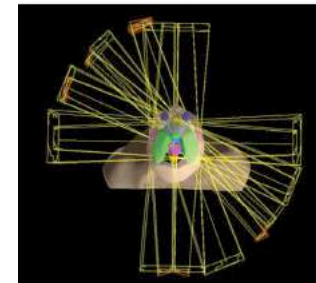
Collimation system



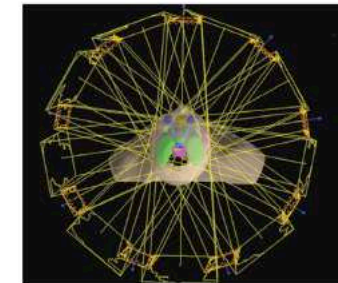
(a)



(b)



(c)



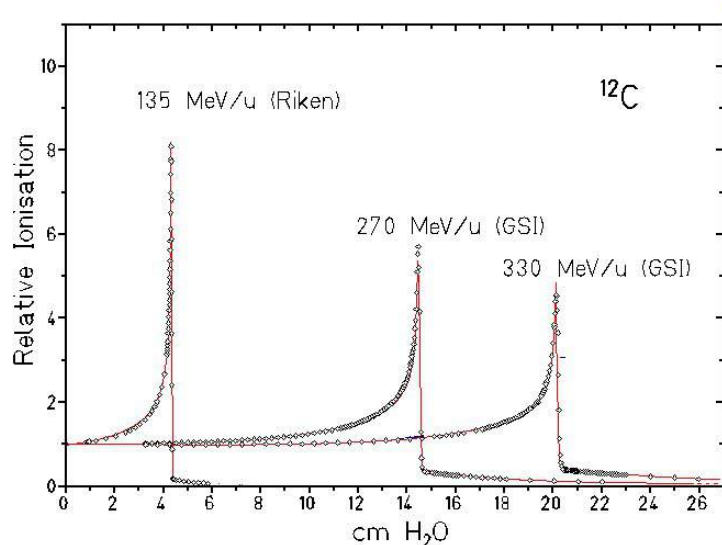
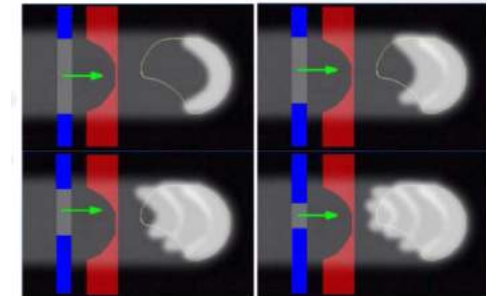
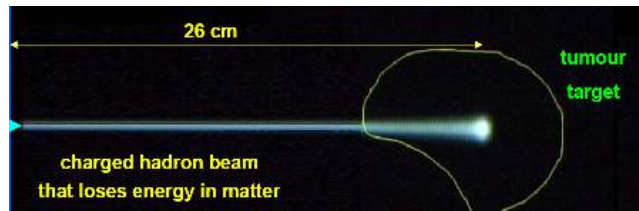
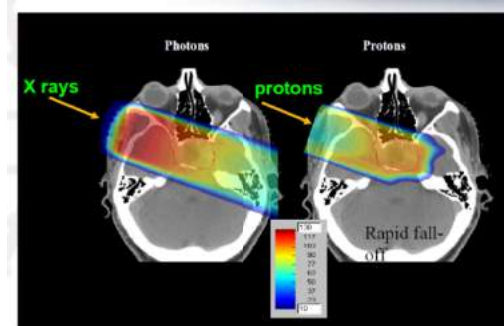
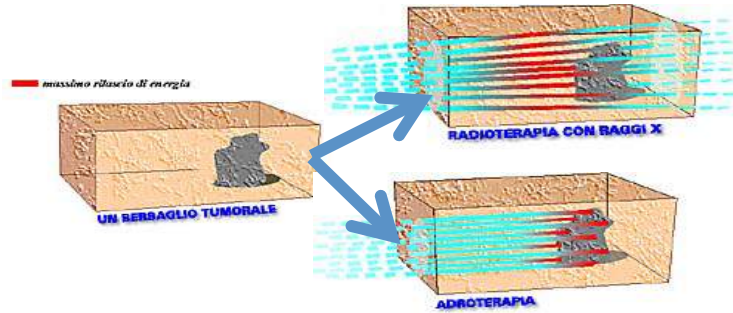
(d)

# MEDICAL APPLICATIONS: ADROTHERAPY

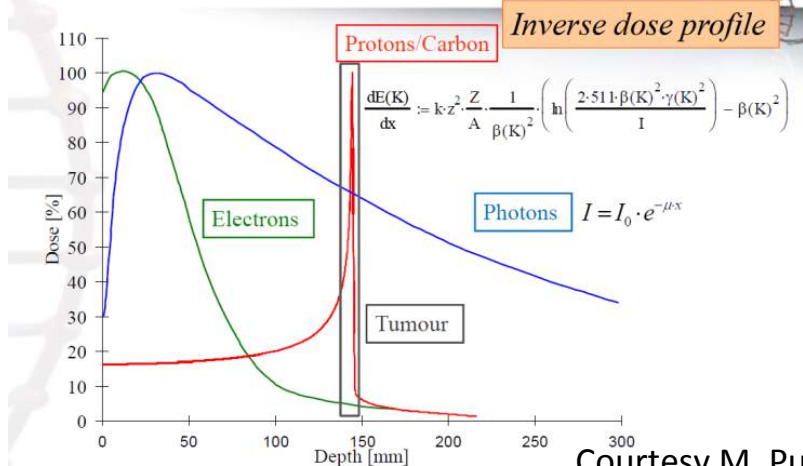
Anti-tumor therapy based on irradiation with protons and heavy ions (C).

It is more effective and more localized (Bragg resonance) than that based on electrons or X-rays

Specialized centers: CNAO in Pavia, PSI in Zurich, Loma Linda in California, Japan, ...



Comparison of the depth dose profiles

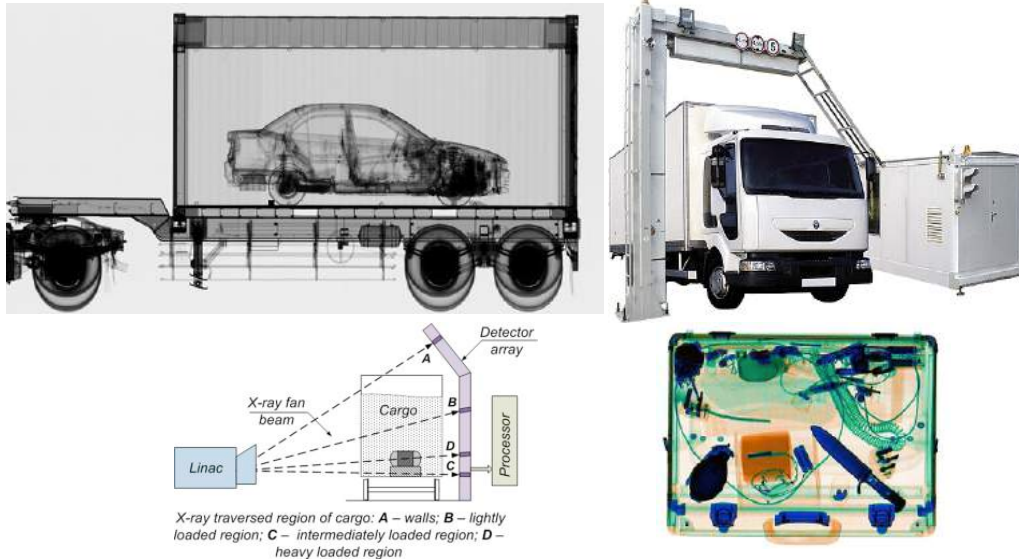


Courtesy M. Pullia



# INDUSTRIAL APPLICATIONS

## Cargo scan with X-rays



## Sterilization and irradiation of food for conservation ("cold pasteurization")

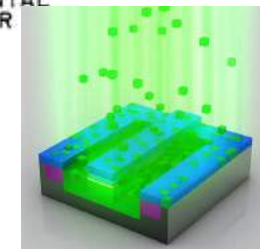
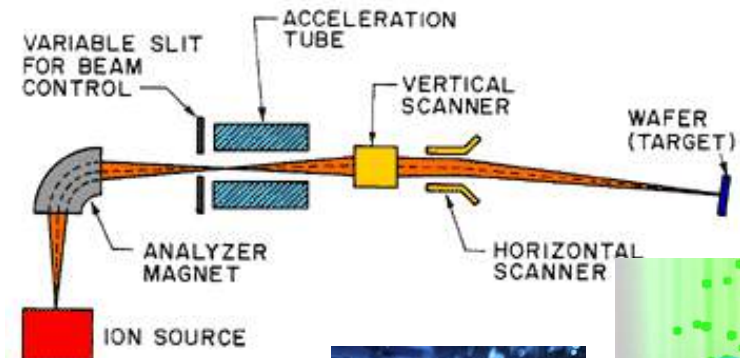


## Treatment of polymeric materials: cross-linking

These industrial treatments increase the performances of the materials in terms of resistance to heat, elasticity, etc ...



## Ion implantation (semiconductors)

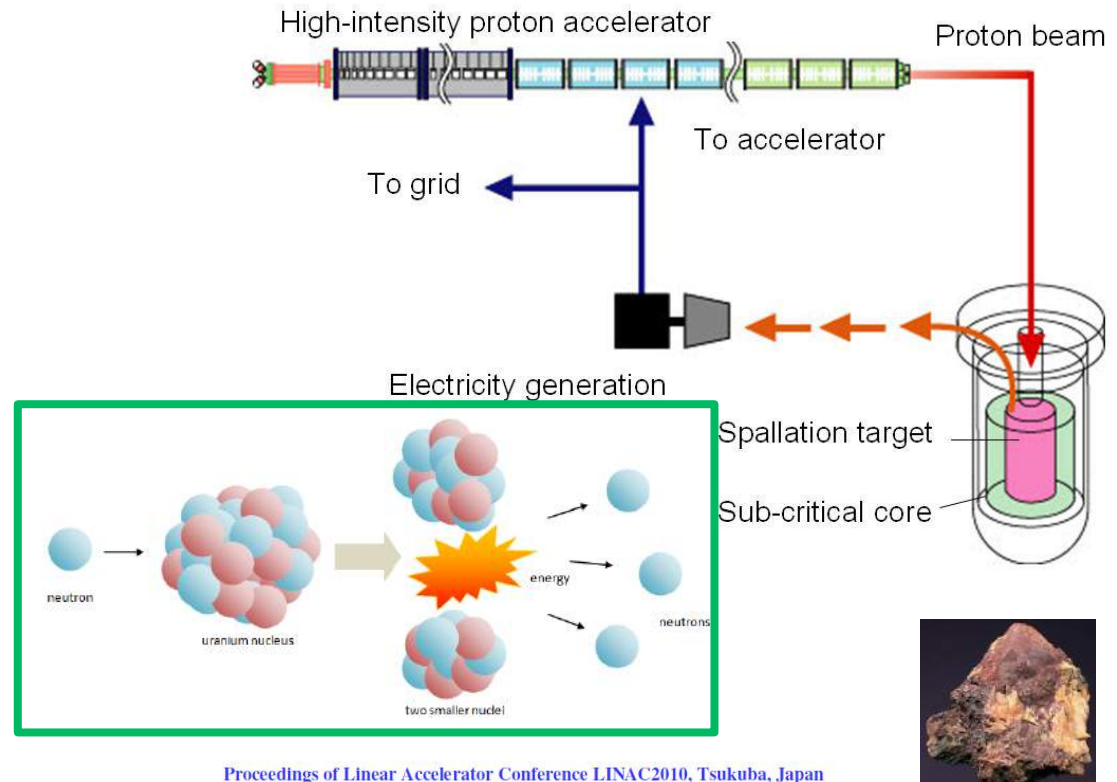


# ENERGY PRODUCTION WITH ACCELERATORS

An **ADS (Accelerator Driven System)** is a sub-critical fission nuclear reactor driven by a high-energy **proton accelerator** (600 MeV-1GeV). The neutrons needed to sustain the fission process are provided by the particle accelerator

Advantages:

- Use **thorium as fuel**, much more abundant than uranium and plutonium
- **short life of waste products** (of the order of 100 years against the hundreds of thousands of years of current reactors).
- intrinsically safe reaction (**controlled fission**)



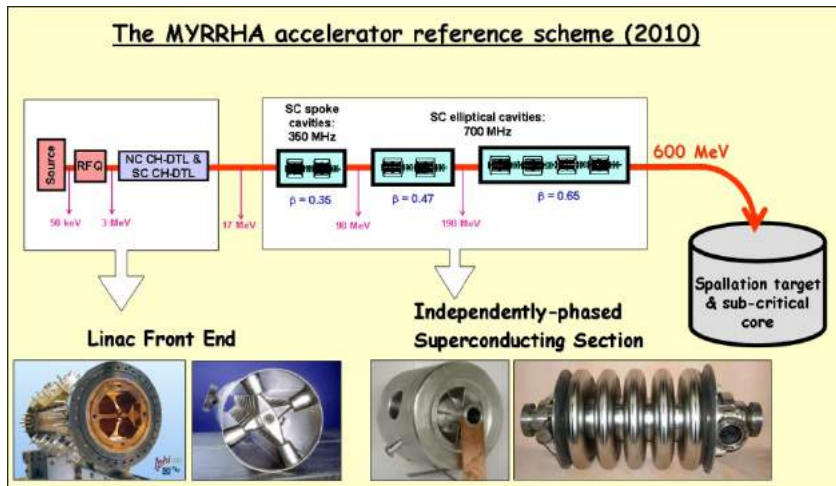
Proceedings of Linear Accelerator Conference LINAC2010, Tsukuba, Japan

## PROTON LINAC FOR ADS APPLICATION IN CHINA

Shinian Fu, Shouxian Fang, Jiuqing Wang  
 IHEP, Institute of High Energy Physics, Beijing 100049, China  
 Xialing Guan  
 CIAE, China Institute of Atomic Energy, Beijing 102413, China

Some laboratory experiments and many theoretical studies have demonstrated the feasibility of this plant. **Carlo Rubbia**, was one of the first to conceive a project of a subcritical reactor, the so-called "**energy amplifier**".

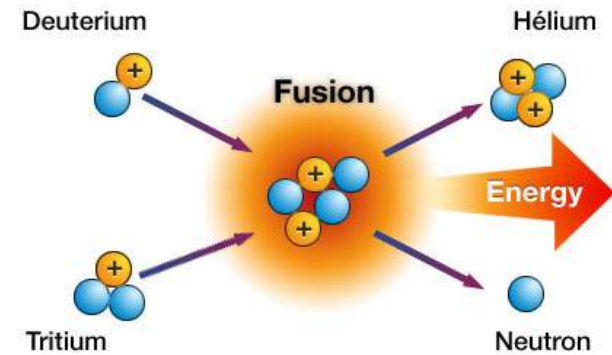
In 2012, CERN engineers launched the International Thorium Energy Committee (iTheC) to promote this goal.



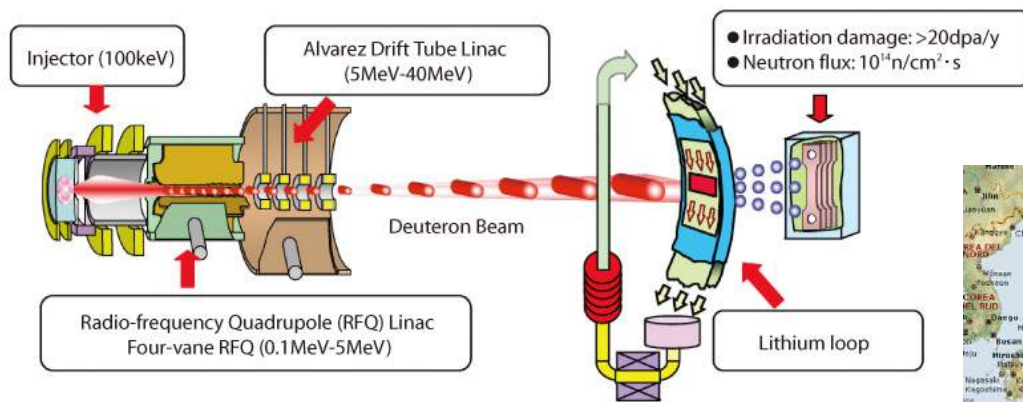
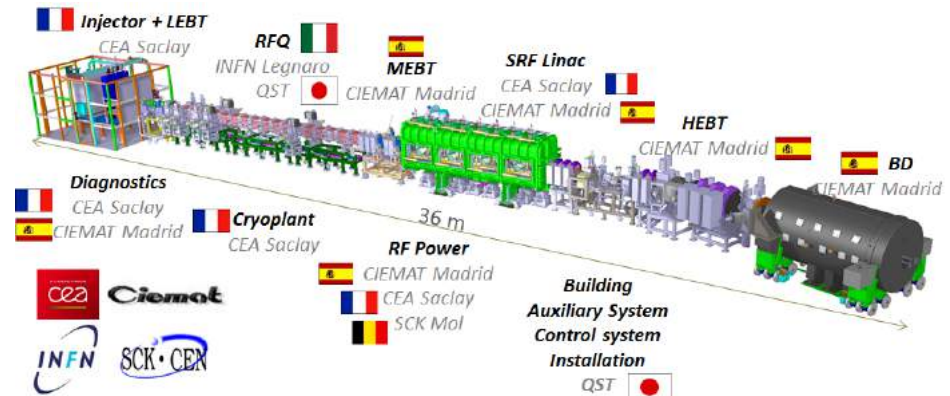


# MATERIALS TESTS FOR NUCLEAR FUSION REACTORS

In a **future nuclear fusion reactor** the generated **neutron flux** is of the order of  $10^{18} \text{ m}^{-2}\text{s}^{-1}$  with an energy of 14.1 MeV that collide against the inner walls of the reactor itself

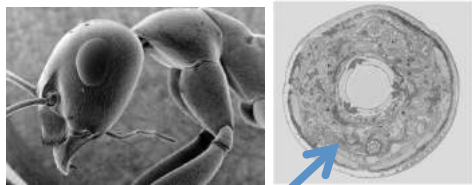


The **International Fusion Materials Irradiation Facility (IFMIF)** is a test facility for testing materials that can be used in a fusion reactor. It is a **neutron source** based on a deuterium accelerator which colliding against the lithium atoms produces a flow of neutrons similar to that foreseen in the first wall of a fusion reactor.



# EXPLORING MATTER WITH SMALL WAVELENGTH

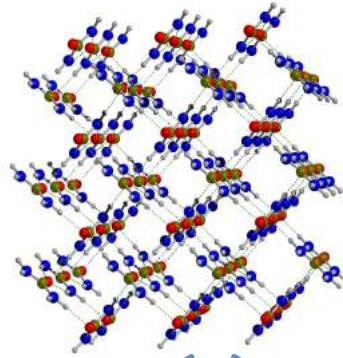
Visible light  
400-700 nm ( $\sim 10^{-7}$  m)



1-10  $\mu$ m

Resolution 200 nm

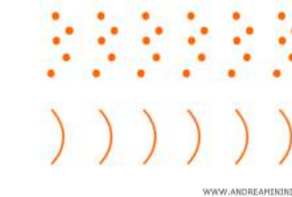
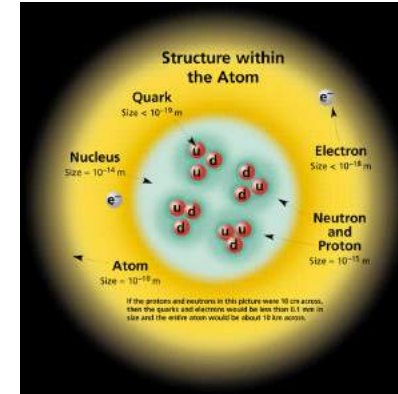
X Rays:  
0.01-10 nm ( $\sim 10^{-11}$ - $10^{-8}$  m)



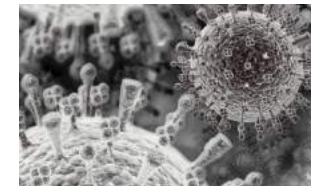
$\sim 10^{-10}$  m



Accelerated particles  
<0.01 nm ( $< 10^{-11}$  m)



$$\lambda = h/p$$

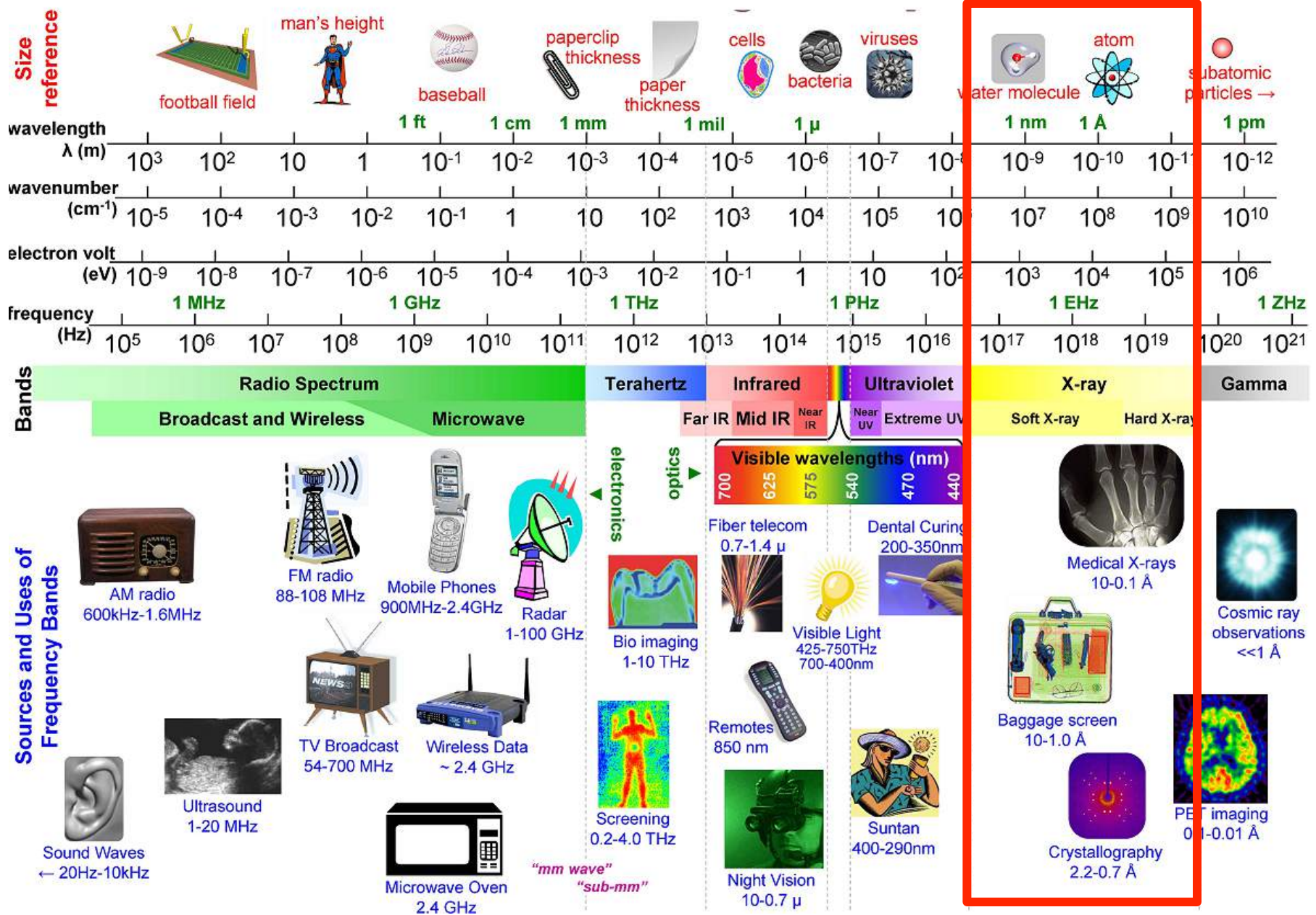


PHOTONS

PARTICLES IN GENERAL

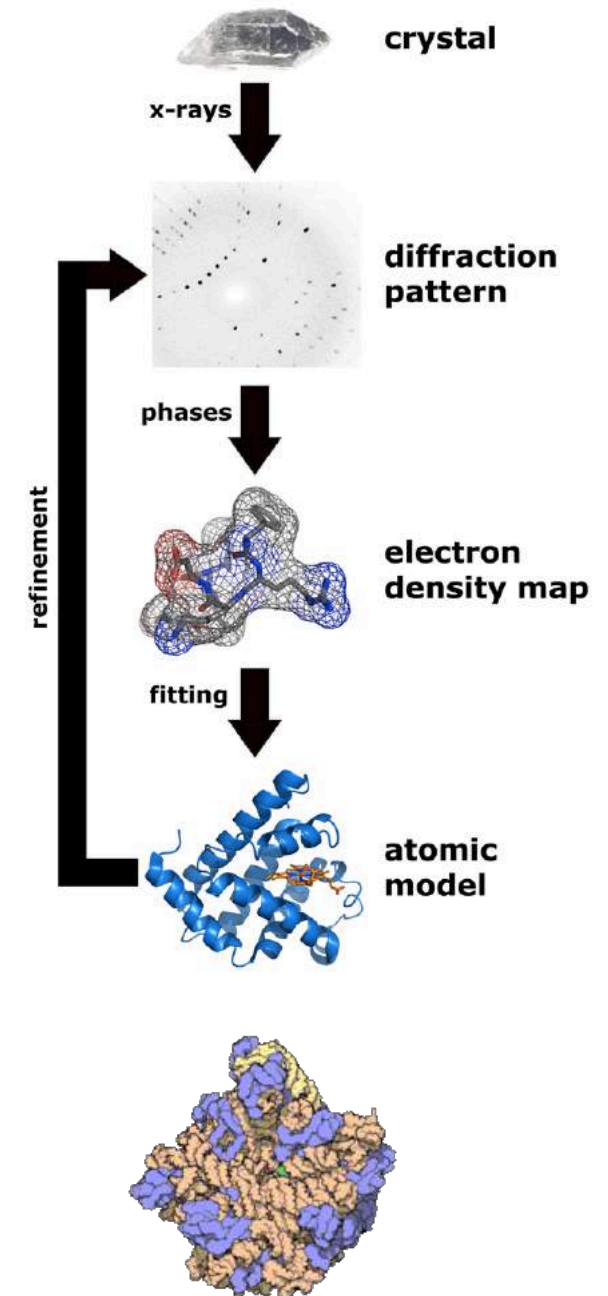
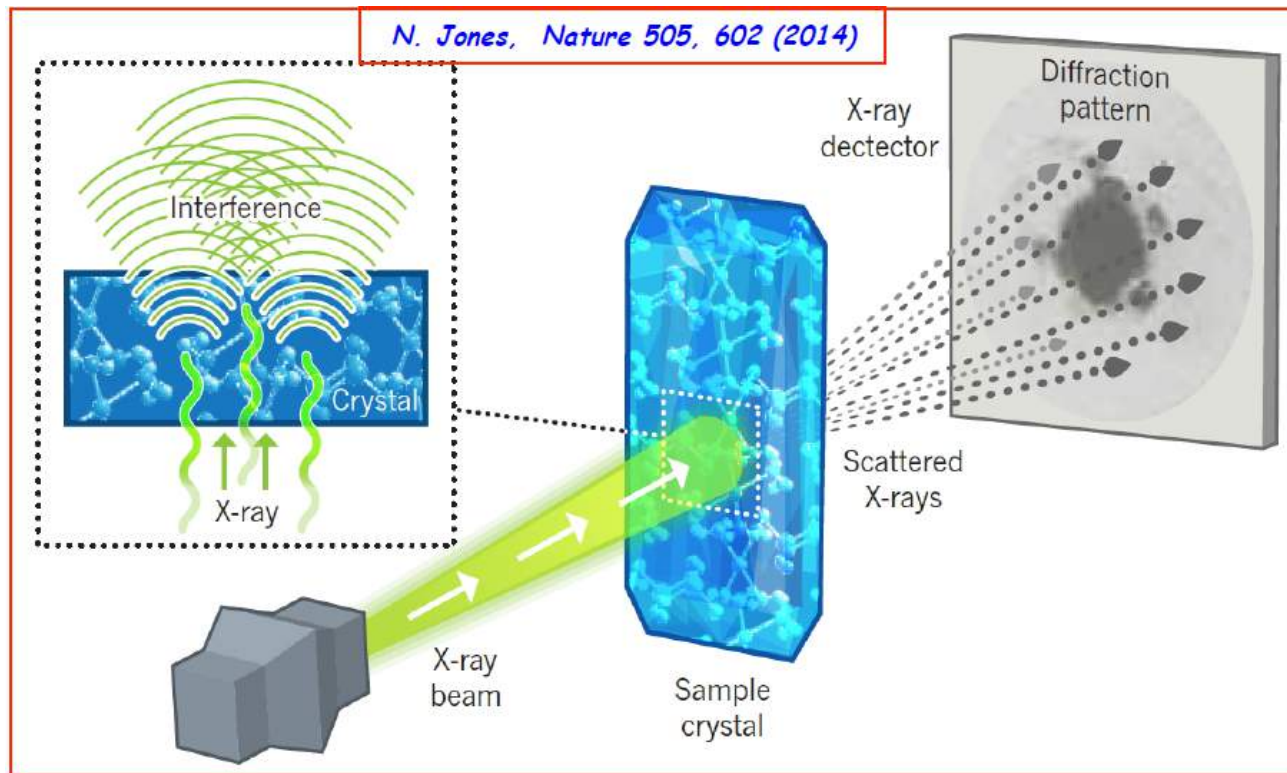


# PHOTONS WITH DIFFERENT WAVELENGTH: X RAYS



# WHY X-RAYS ARE SO IMPORTANT?

Using **X-rays** we can study the atomic structure of materials because **wavelengths are of the order of  $10^{-10}$  m ( $1\text{\AA}$ )**.



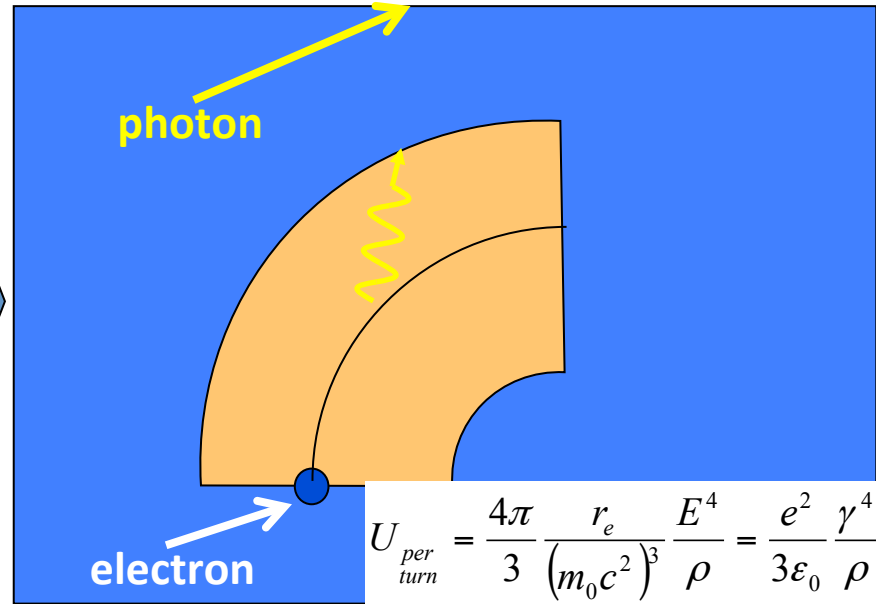
Courtesy A. Balerna



# CAN ACCELERATED PARTICLES EMIT X-RAYS?

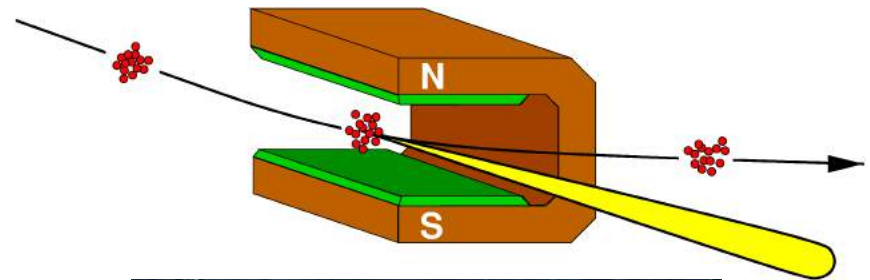
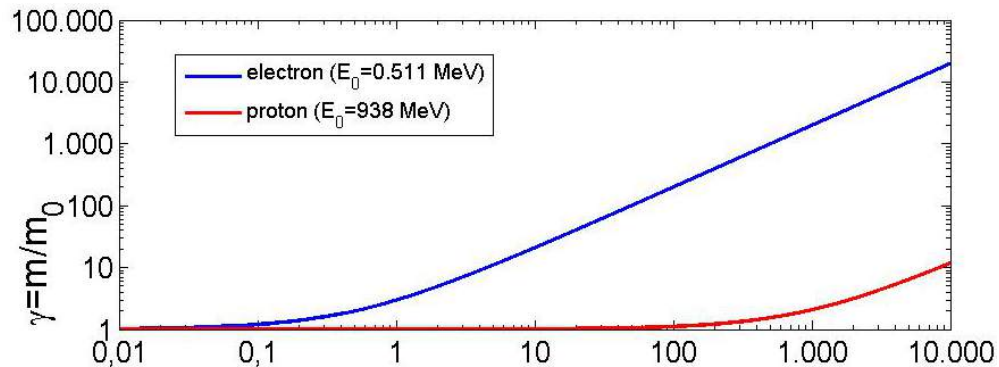
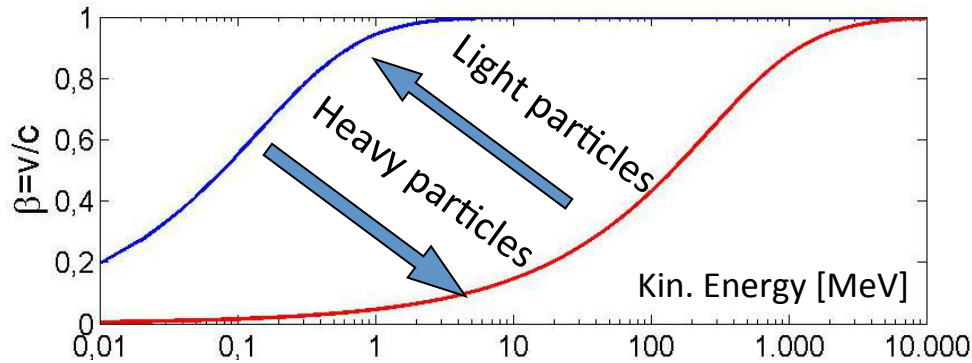
A **charged particle** at a given energy which is bent by a dipole magnet emits electromagnetic radiation (**synchrotron light**).

The radiated energy is related to the fourth power of the **particle relativistic factor**  $\gamma$ . It follows that **only electron machines** (light particles) basically emit photons (except LHC!).

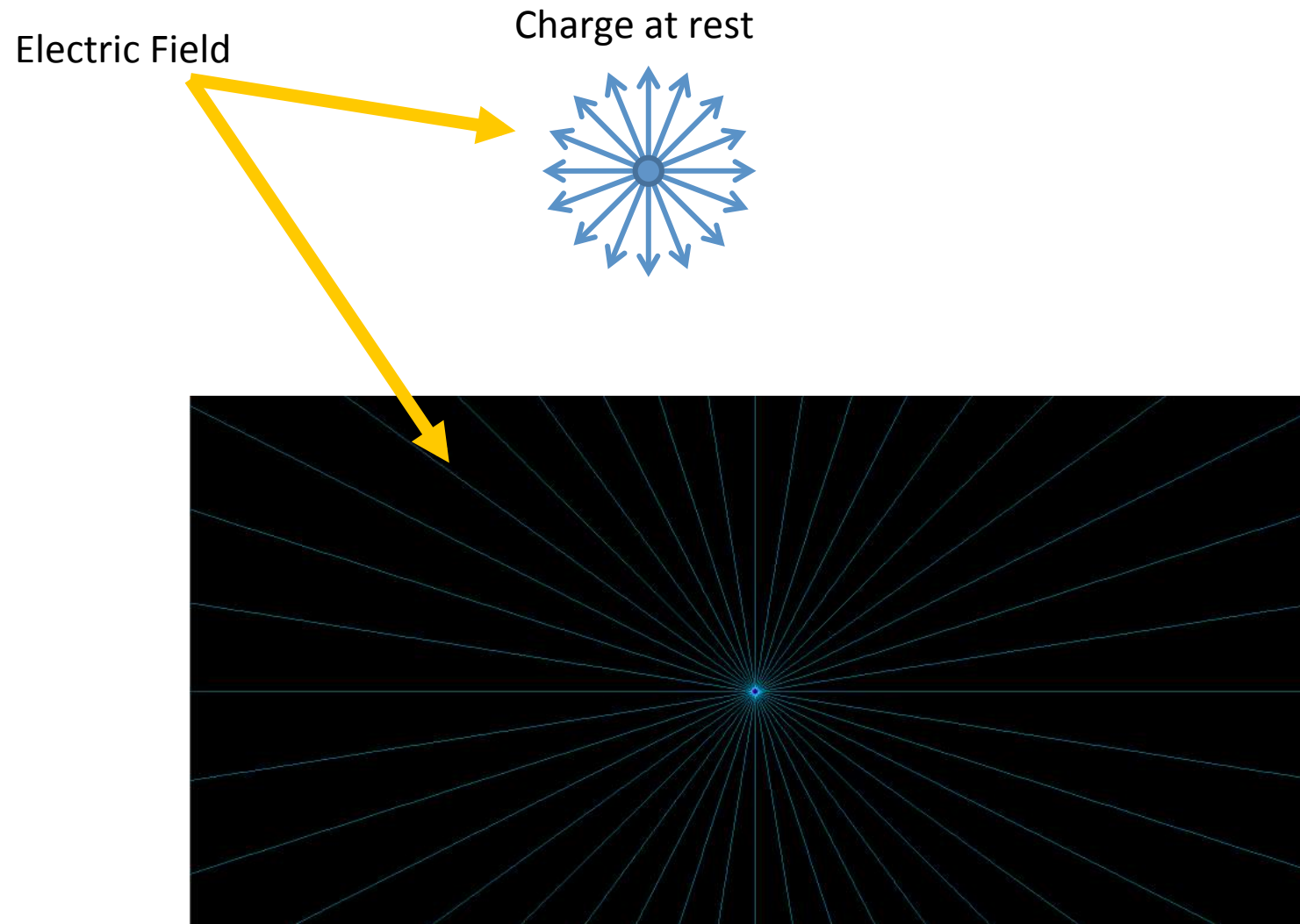


$$\beta = v/c (<1)$$

$$\gamma = E/E_0 (>1)$$

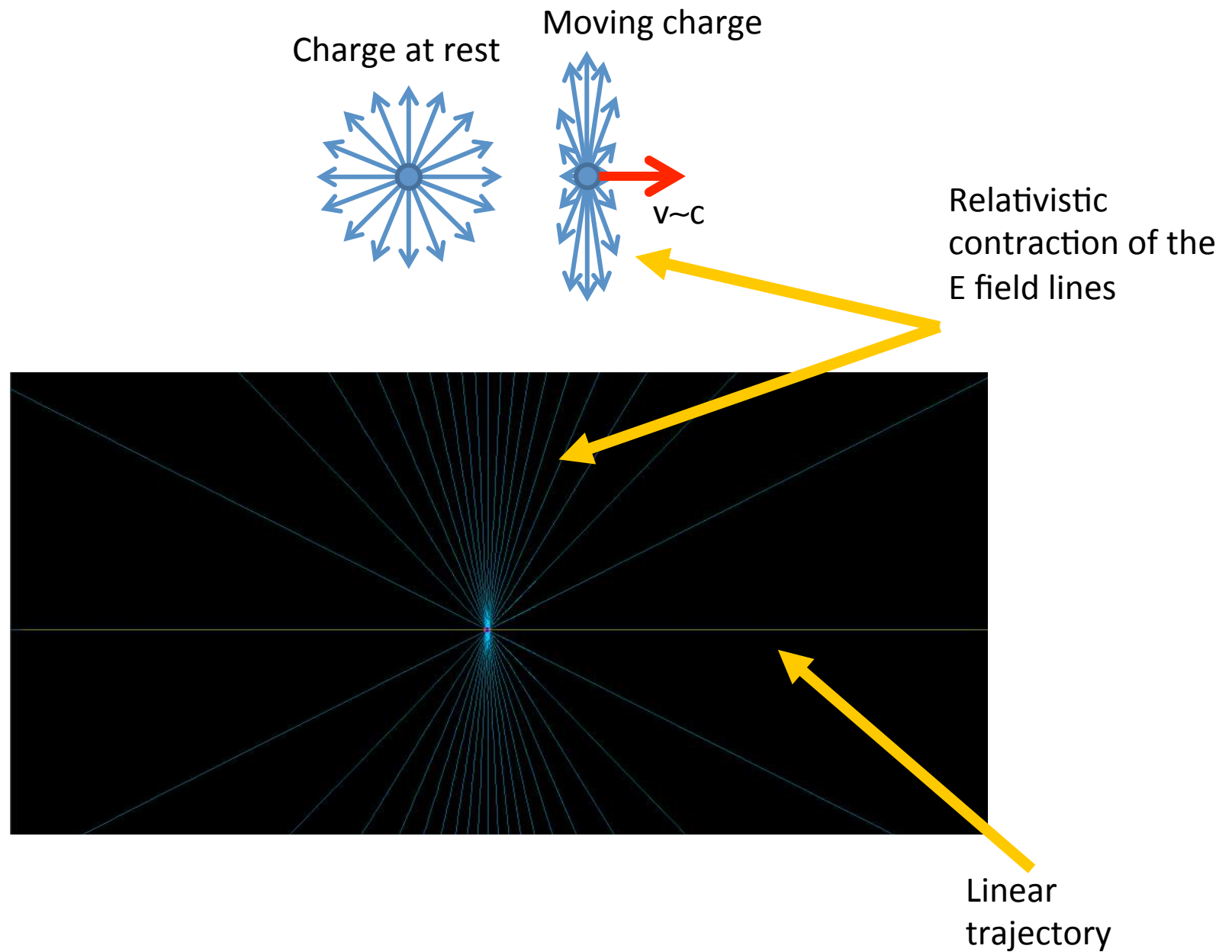


# RADIATION FROM ULTRARELATIVISTIC PARTICLES

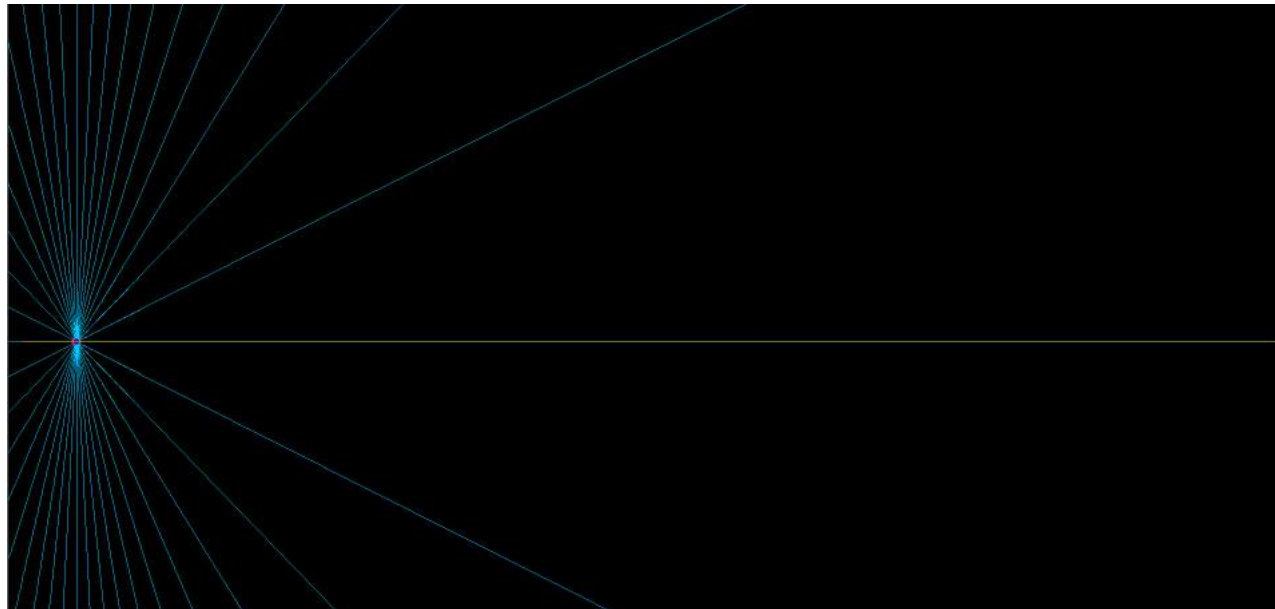




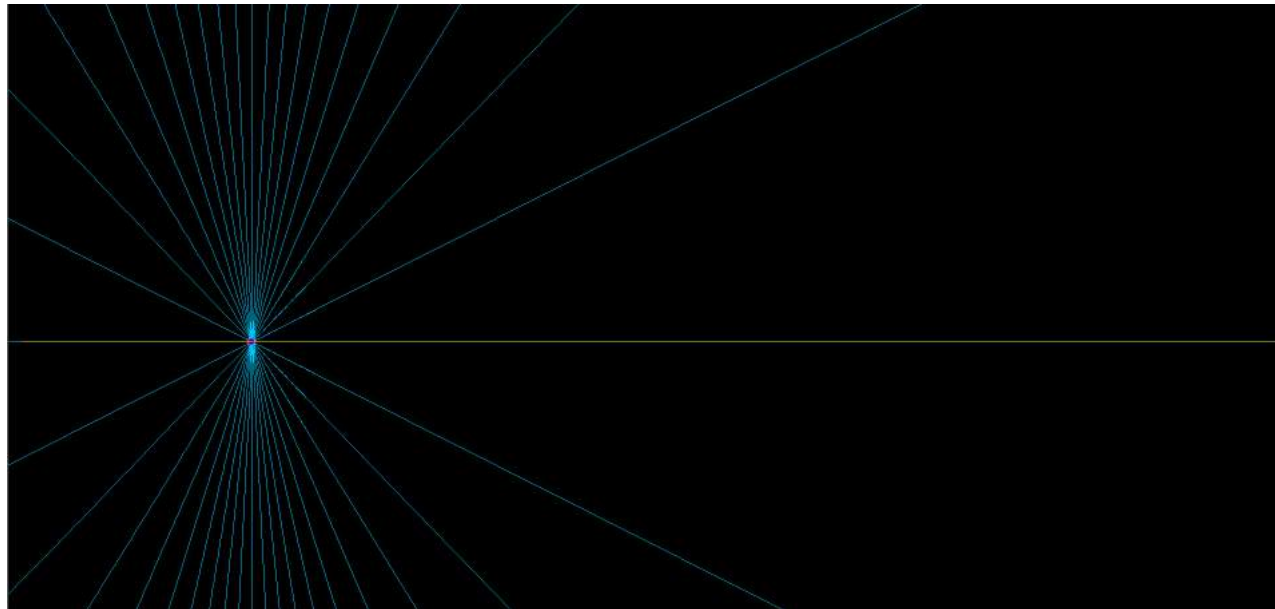
# RADIATION FROM ULTRARELATIVISTIC PARTICLES



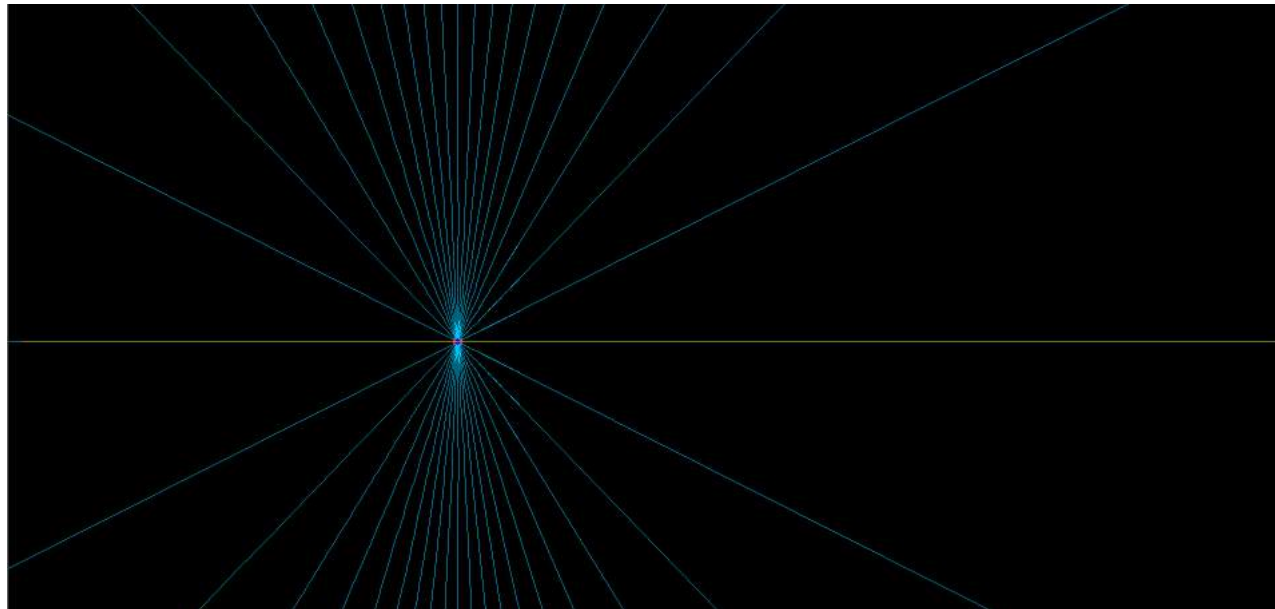
# RADIATION FROM ULTRARELATIVISTIC PARTICLES



# RADIATION FROM ULTRARELATIVISTIC PARTICLES

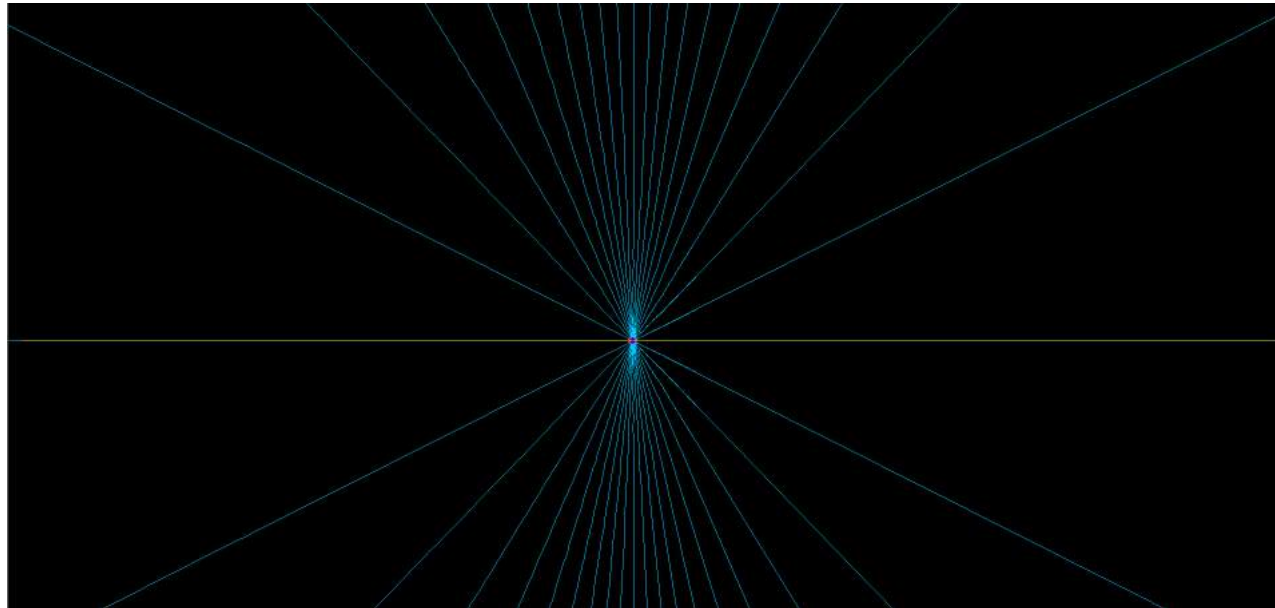


# RADIATION FROM ULTRARELATIVISTIC PARTICLES

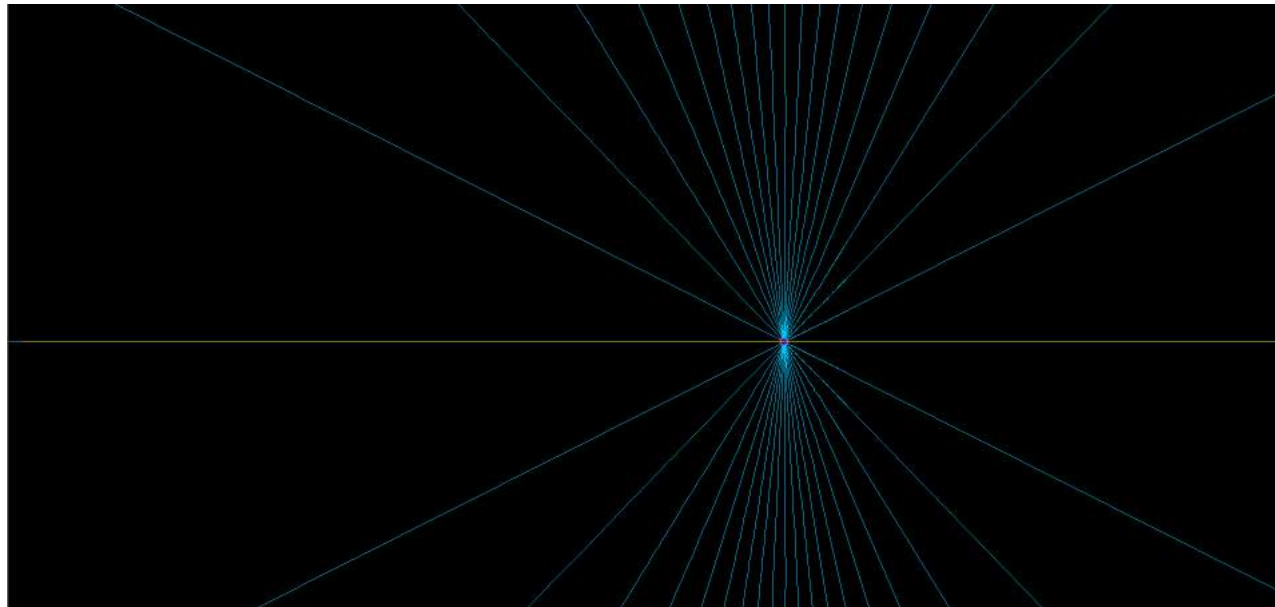




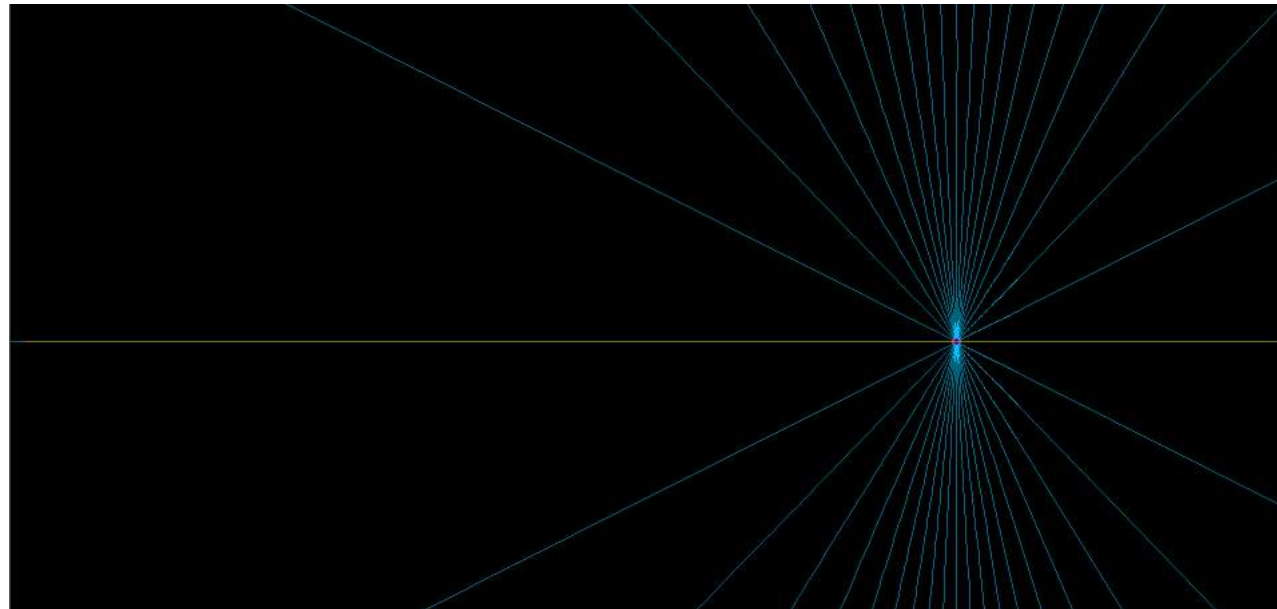
# RADIATION FROM ULTRARELATIVISTIC PARTICLES



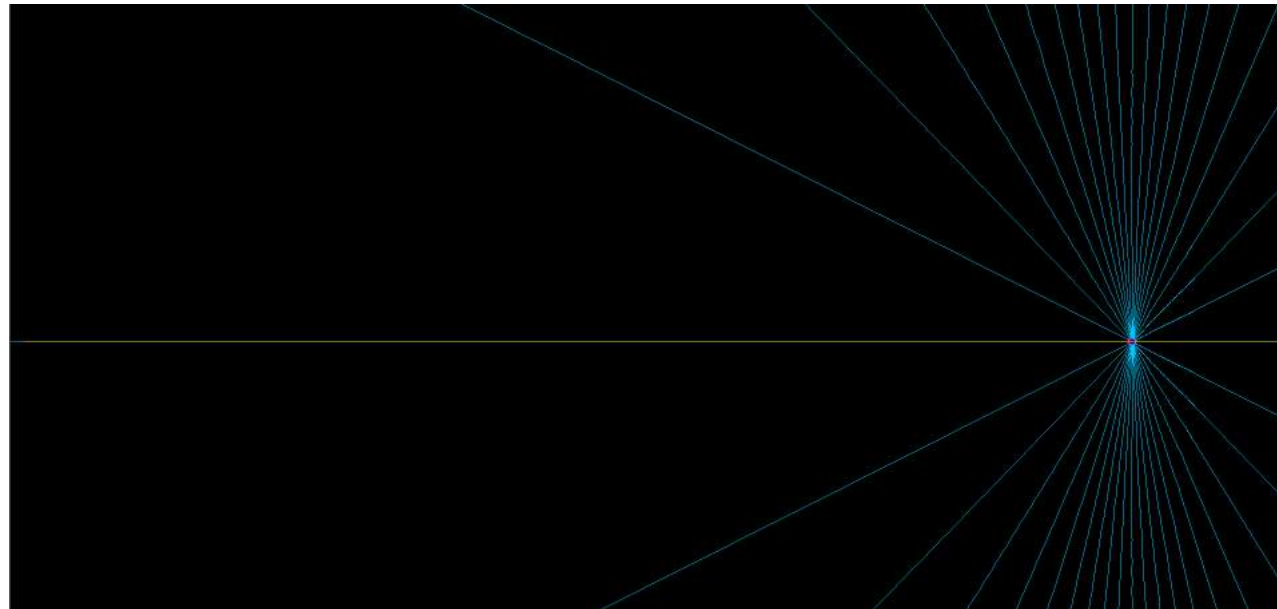
# RADIATION FROM ULTRARELATIVISTIC PARTICLES



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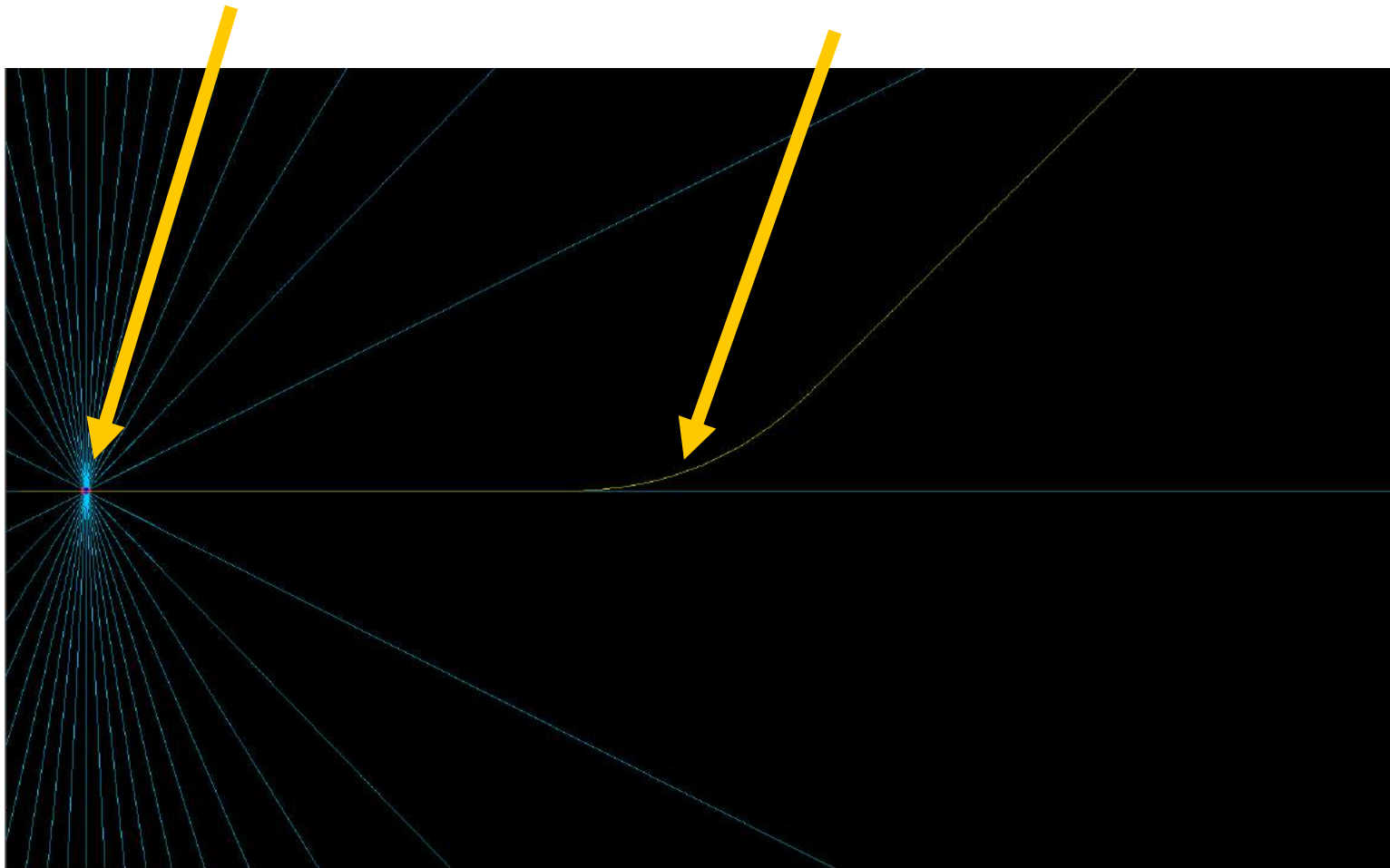




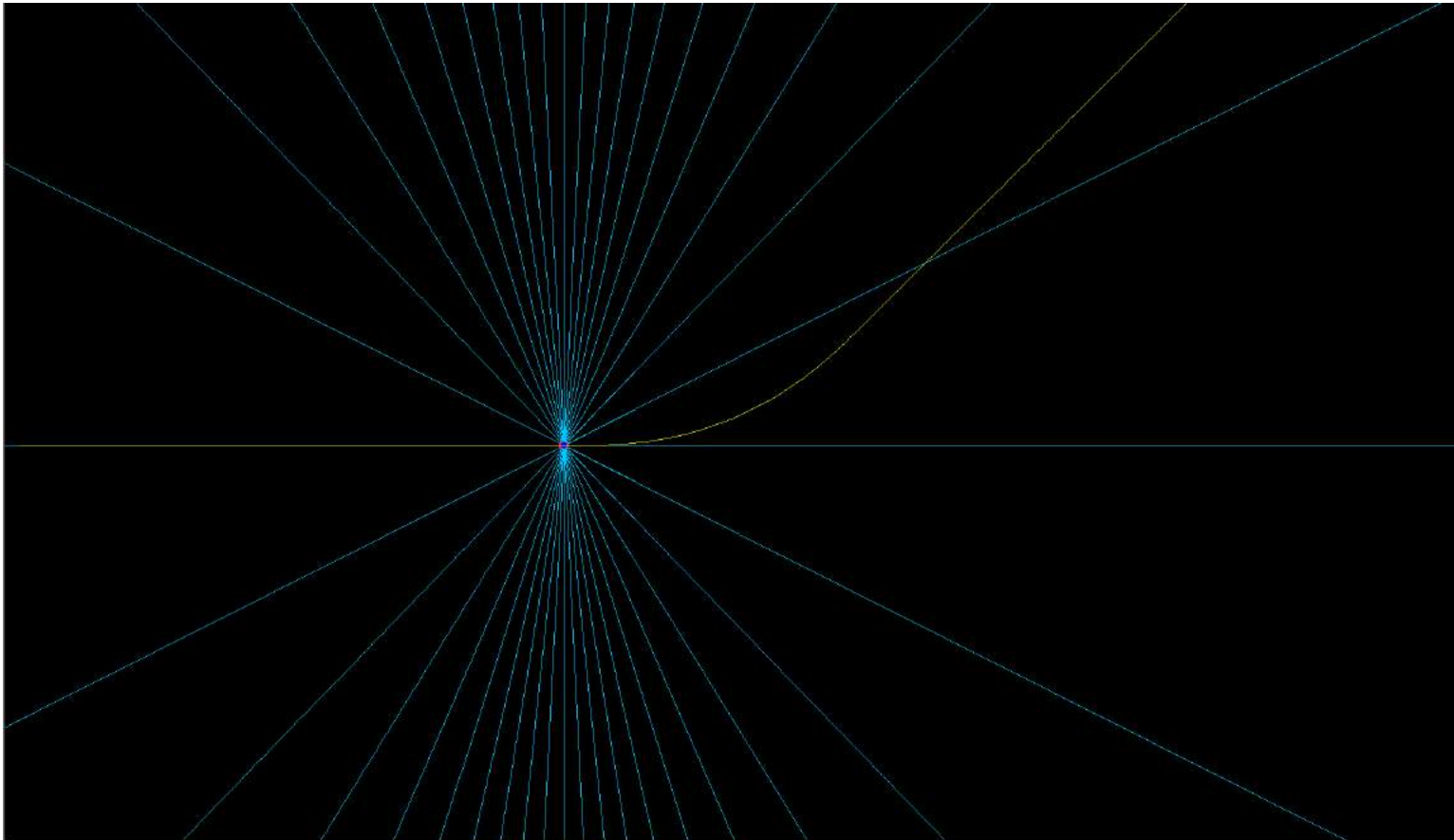
# RADIATION FROM ULTRARELATIVISTIC PARTICLES: DIPOLE

Moving charge

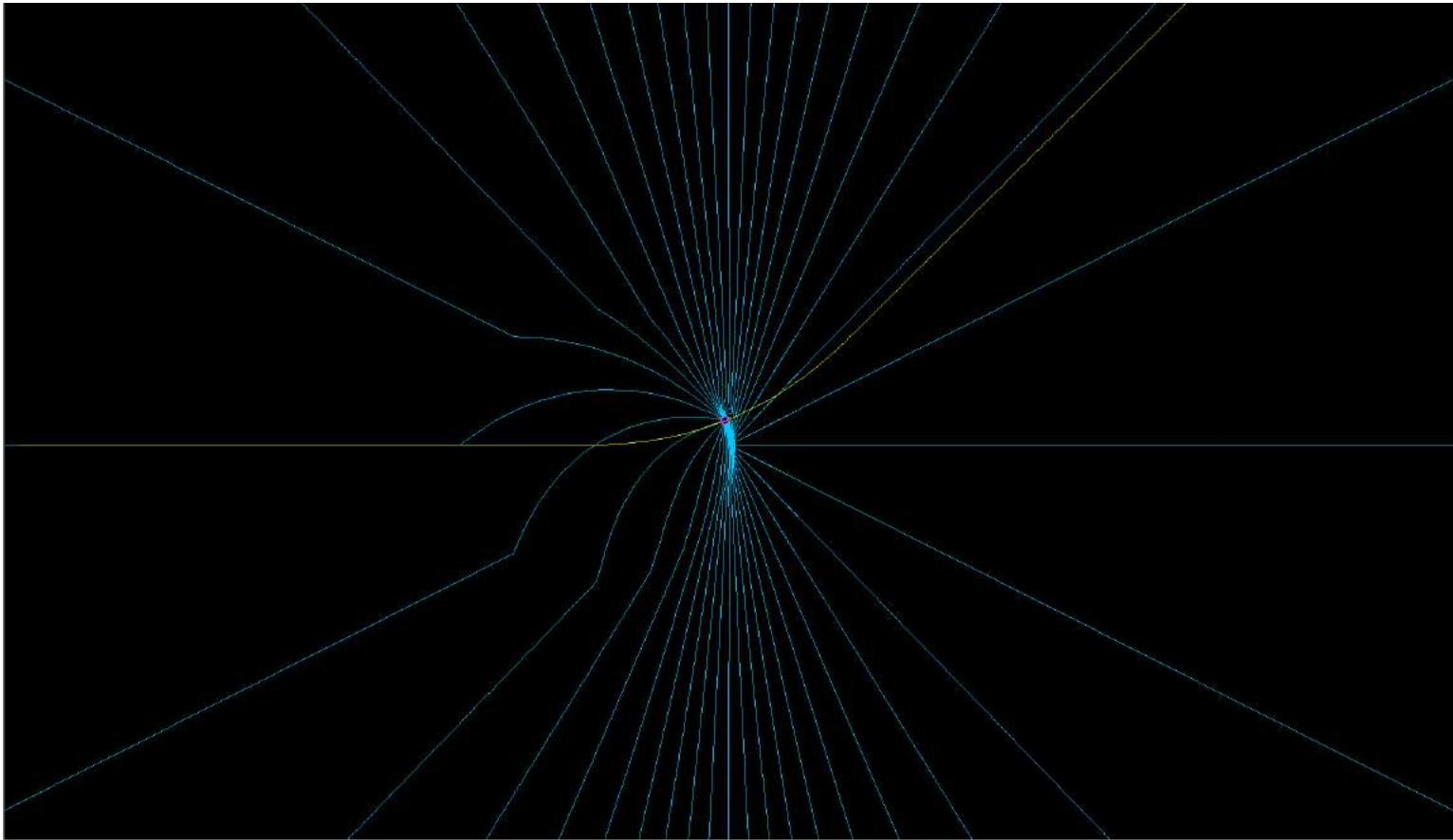
Trajectory from  
a dipole



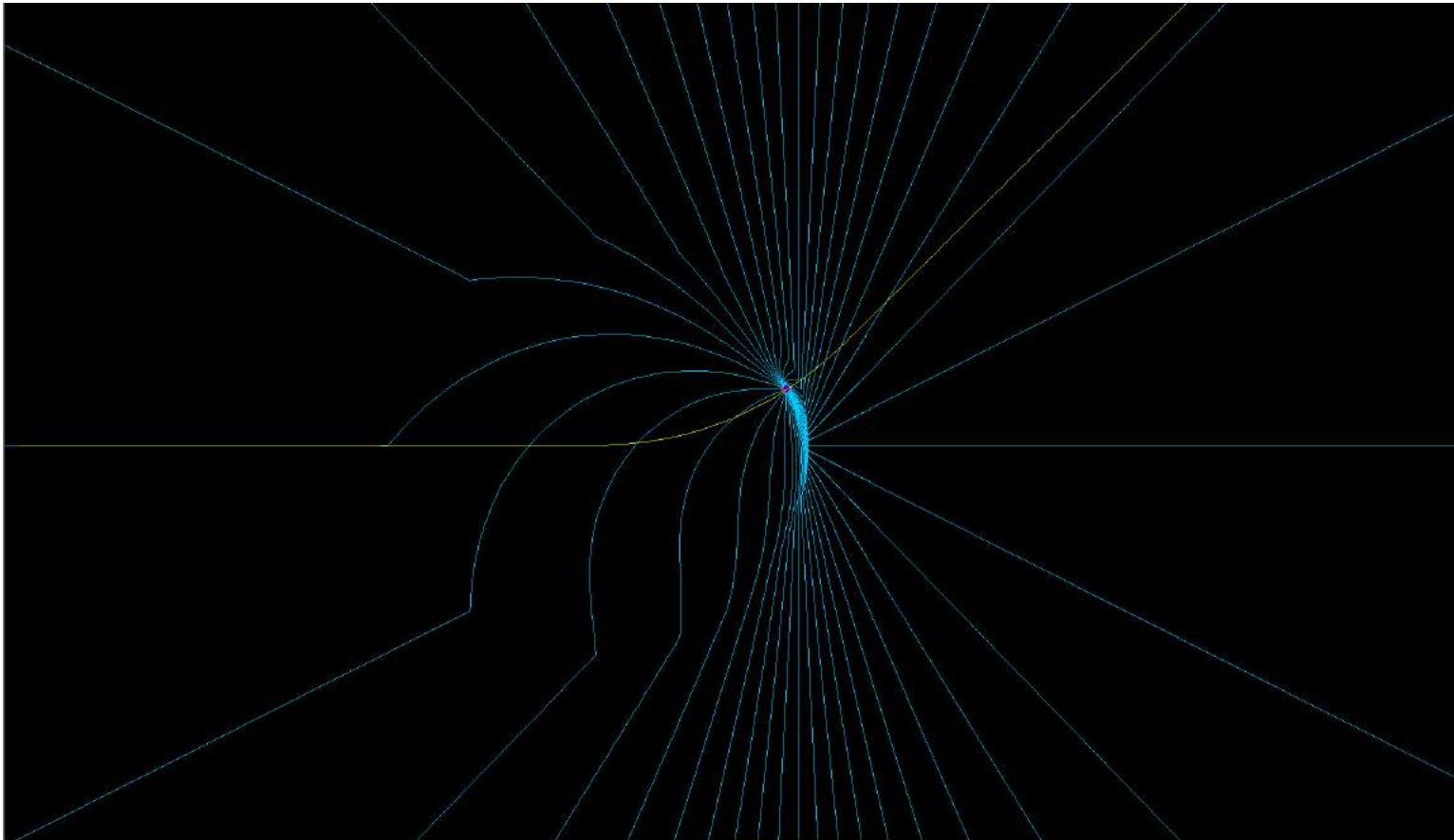
# RADIATION FROM ULTRARELATIVISTIC PARTICLES: DIPOLE



# RADIATION FROM ULTRARELATIVISTIC PARTICLES: DIPOLE

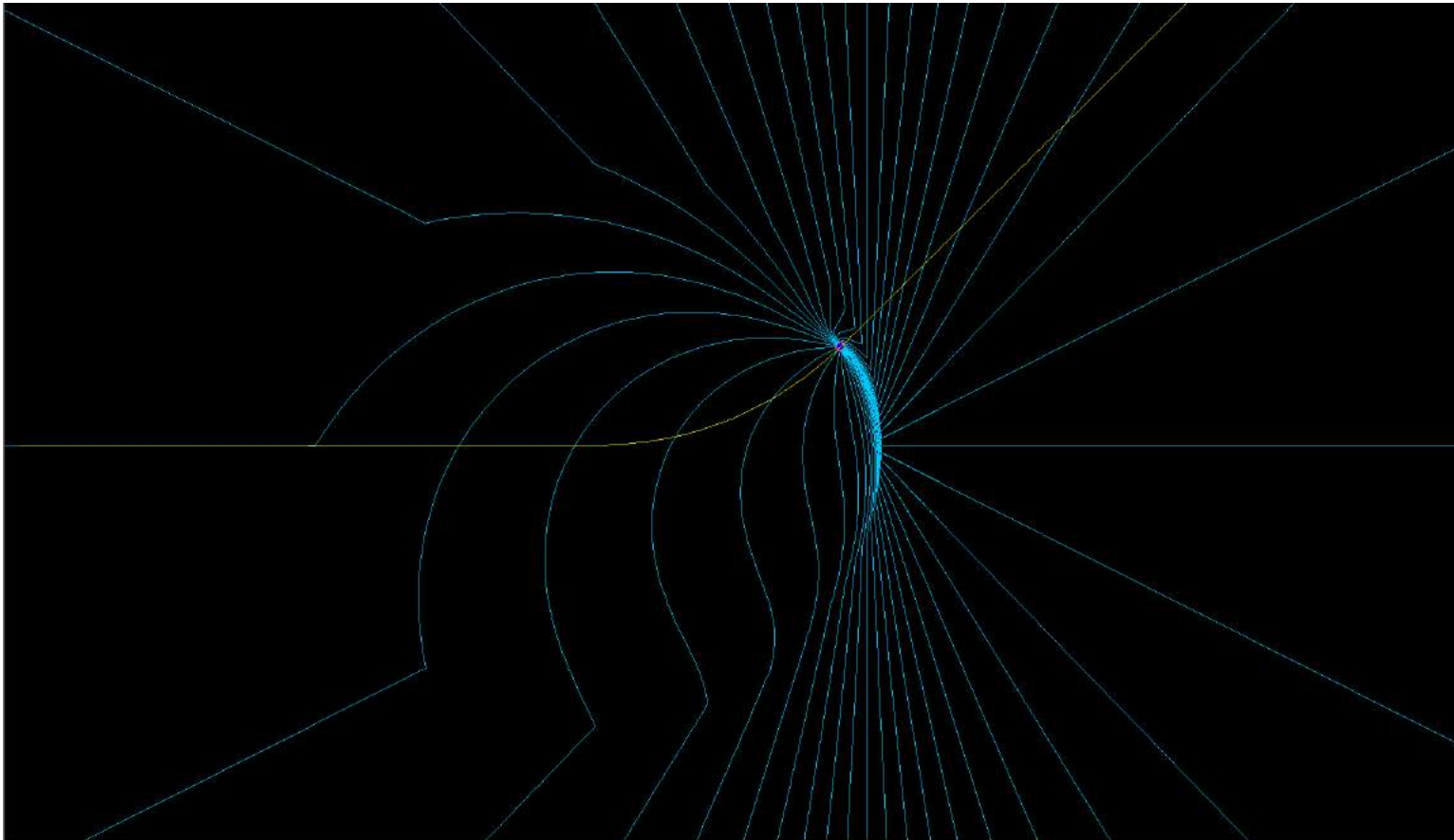


# RADIATION FROM ULTRARELATIVISTIC PARTICLES: DIPOLE

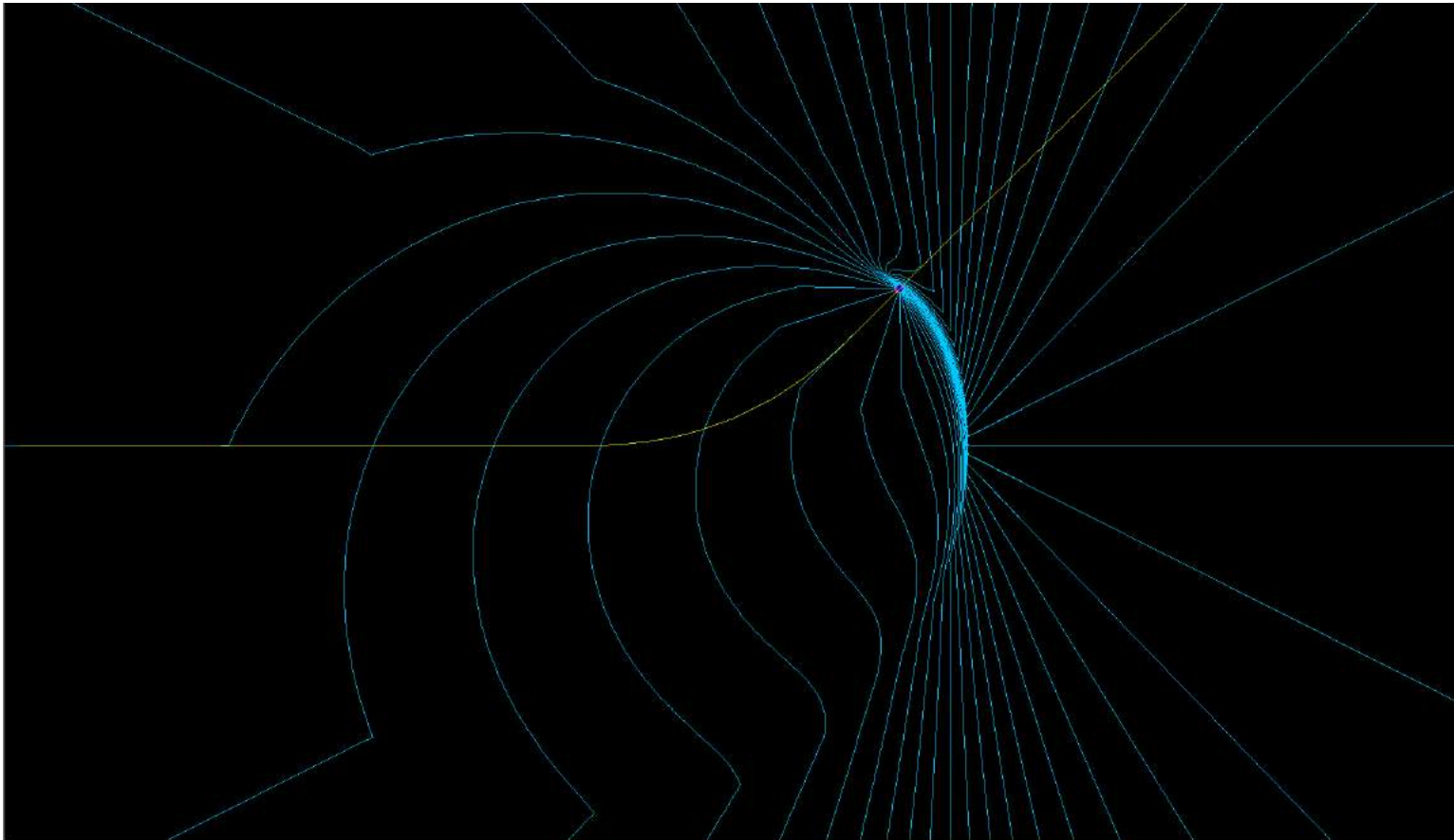




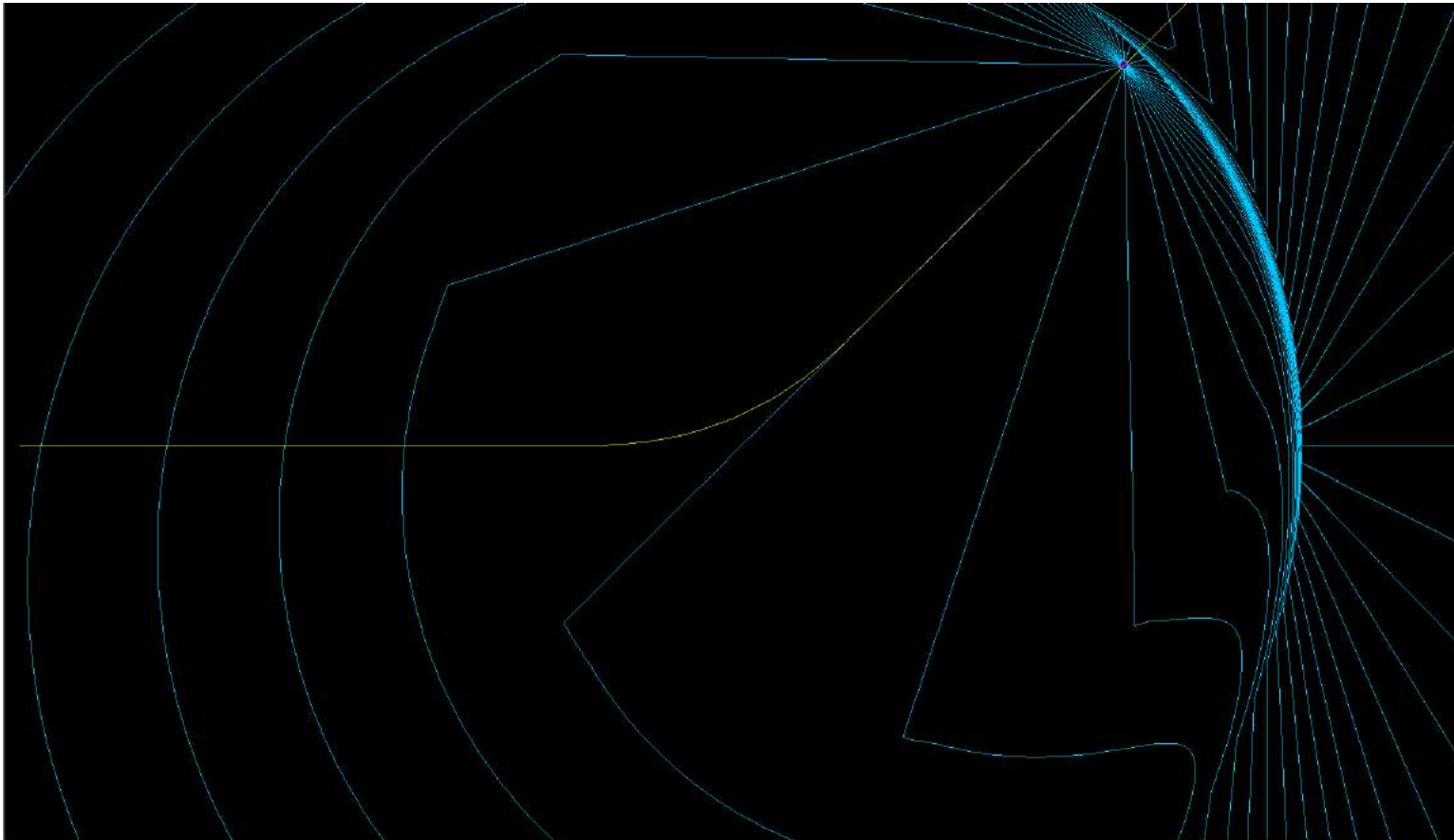
# RADIATION FROM ULTRARELATIVISTIC PARTICLES: DIPOLE



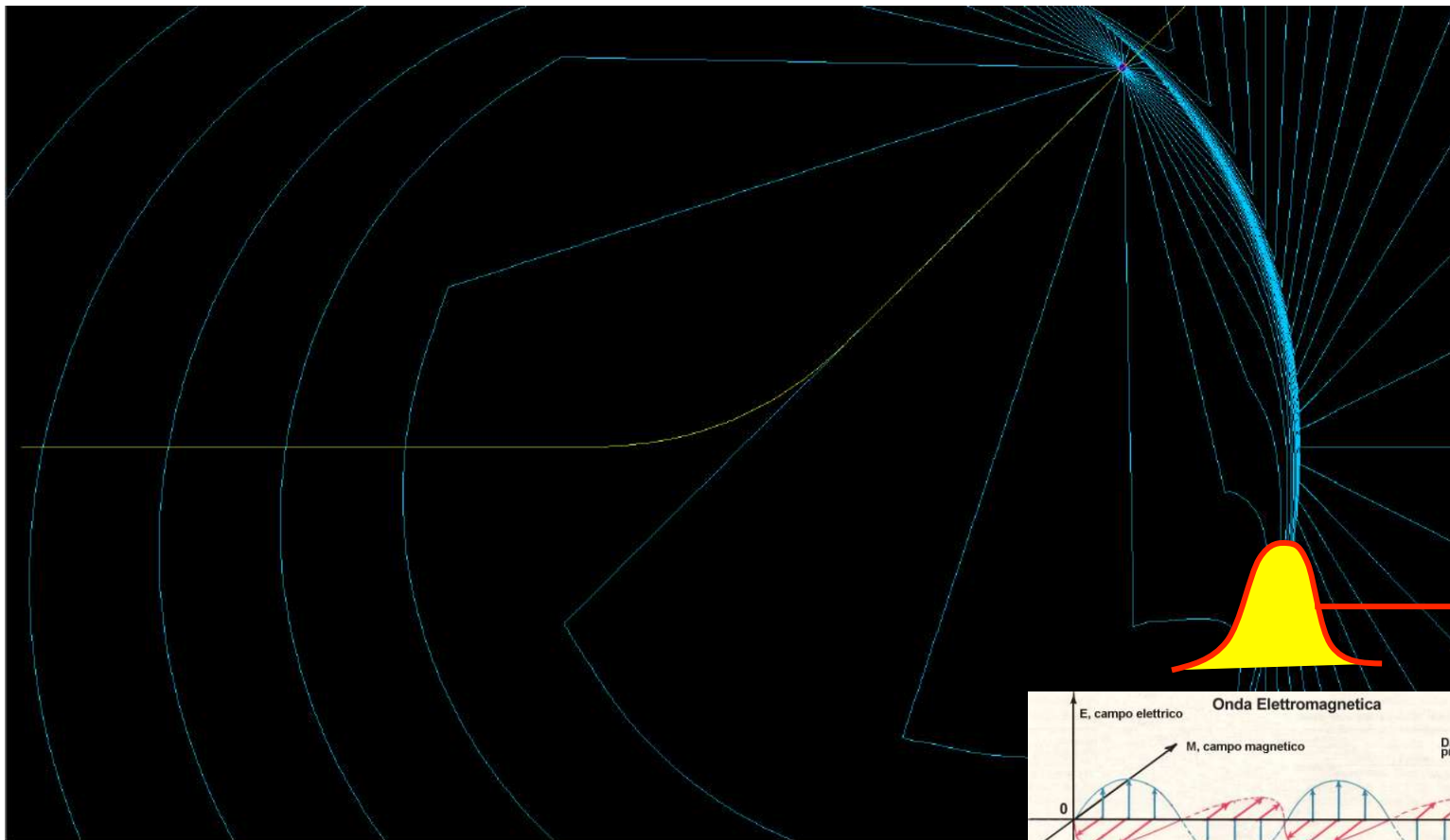
# RADIATION FROM ULTRARELATIVISTIC PARTICLES: DIPOLE



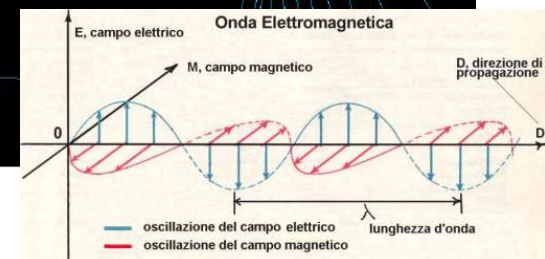
# RADIATION FROM ULTRARELATIVISTIC PARTICLES: DIPOLE



# RADIATION FROM ULTRARELATIVISTIC PARTICLES: DIPOLE



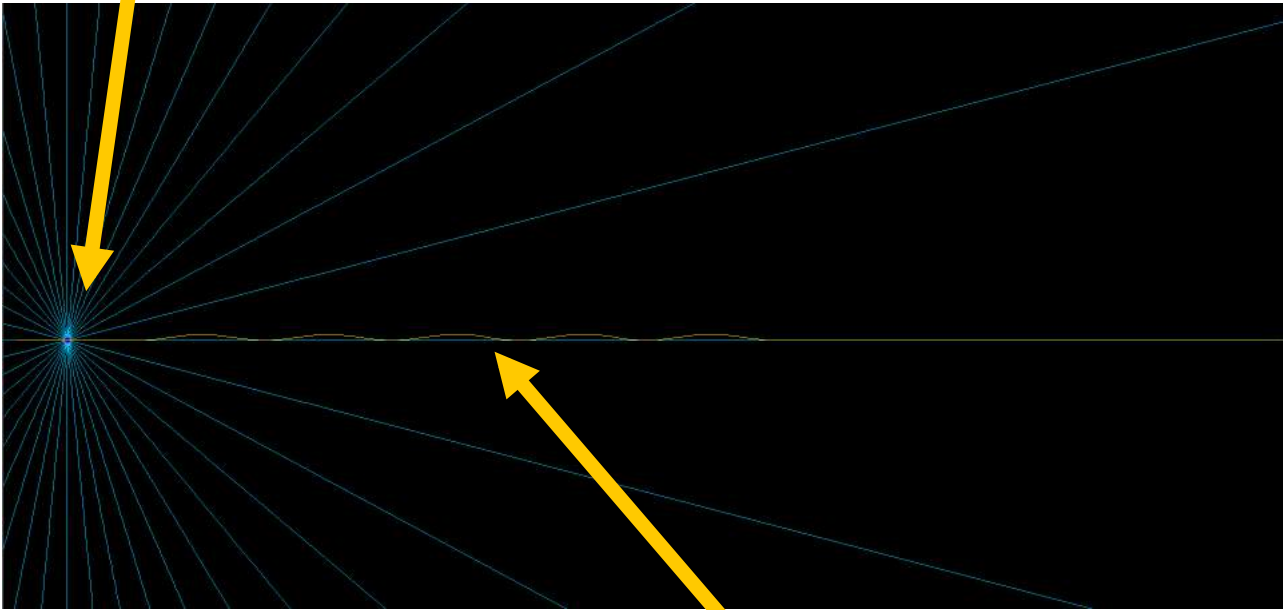
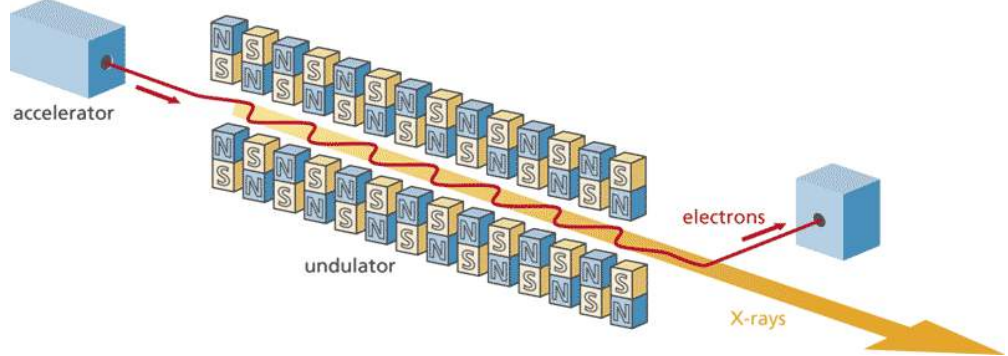
Plane wave





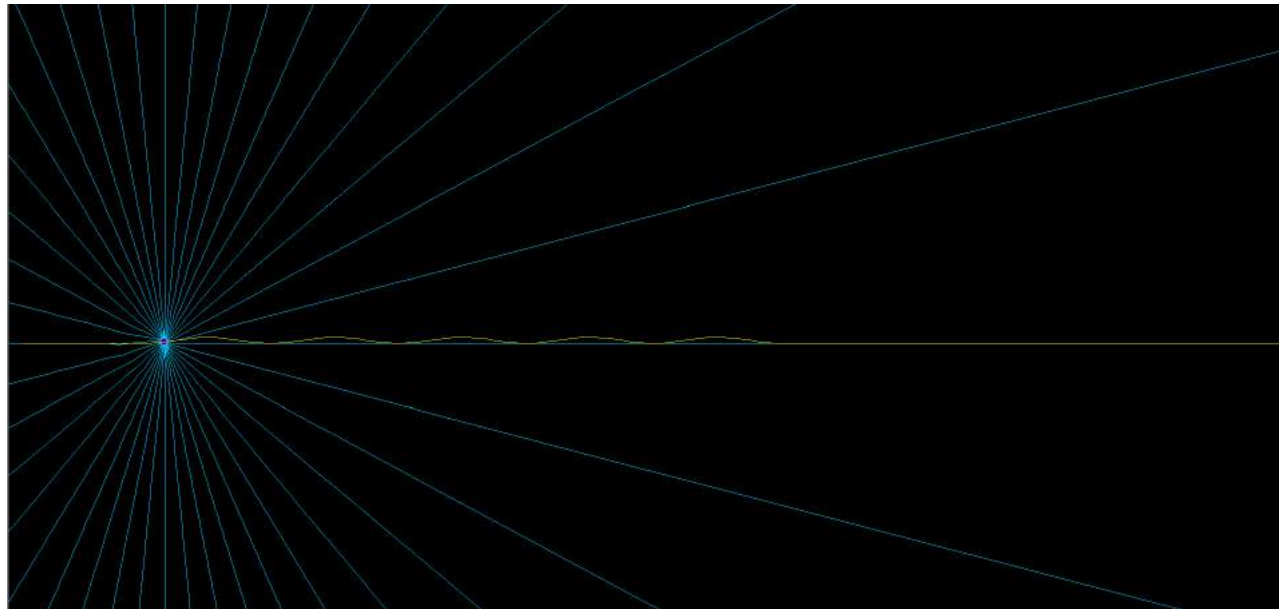
# RADIATION FROM ULTRARELATIVISTIC PARTICLES: UNDULATOR

Moving charge

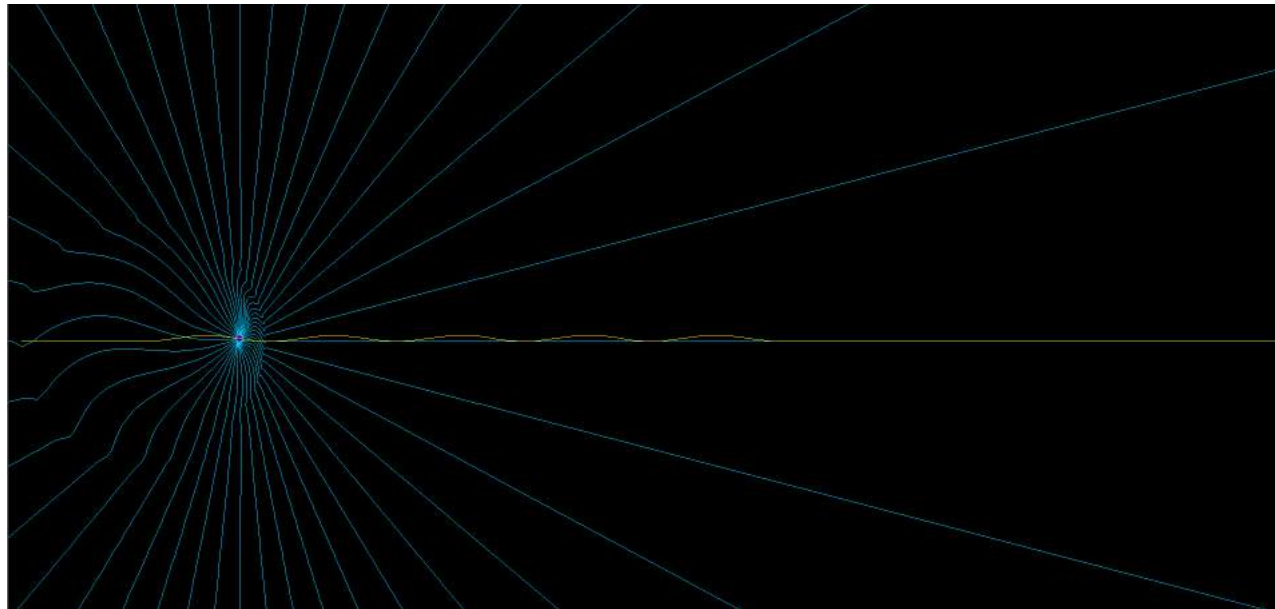


Trajectory in an undulator

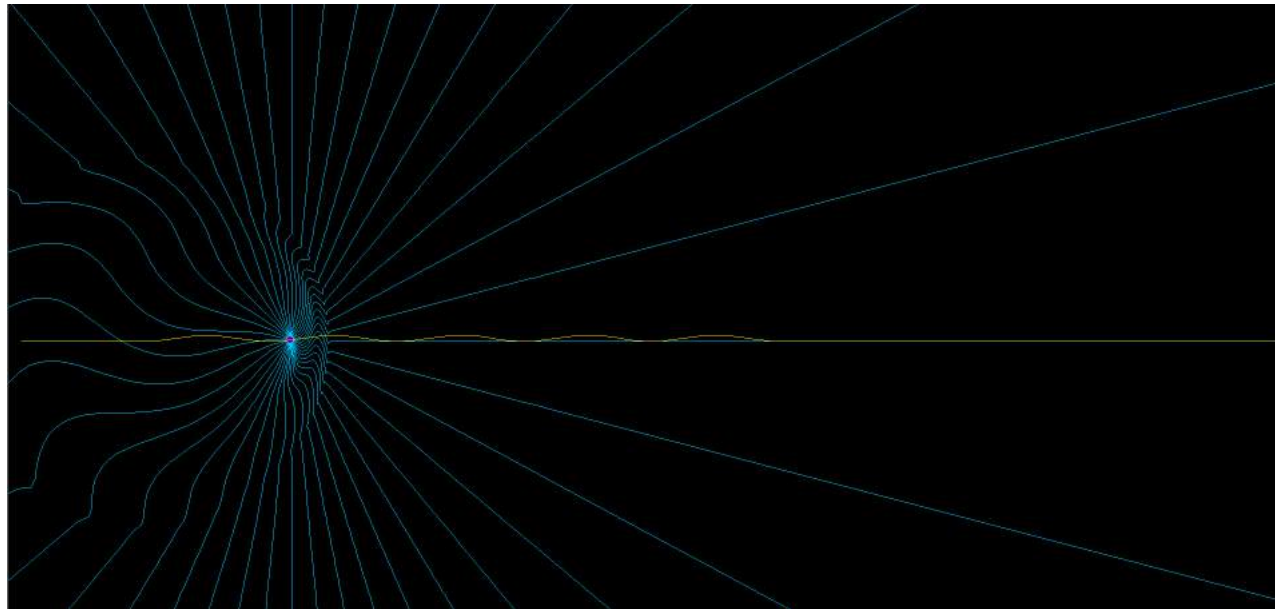
# RADIATION FROM ULTRARELATIVISTIC PARTICLES: UNDULATOR



# RADIATION FROM ULTRARELATIVISTIC PARTICLES: UNDULATOR

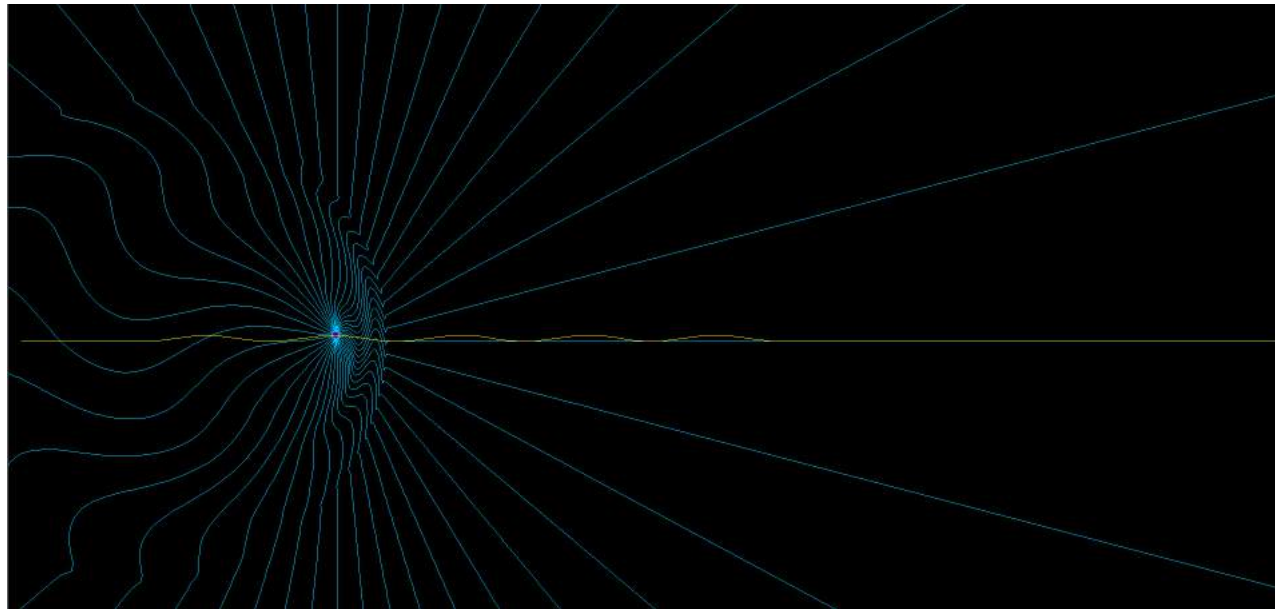


# RADIATION FROM ULTRARELATIVISTIC PARTICLES: UNDULATOR

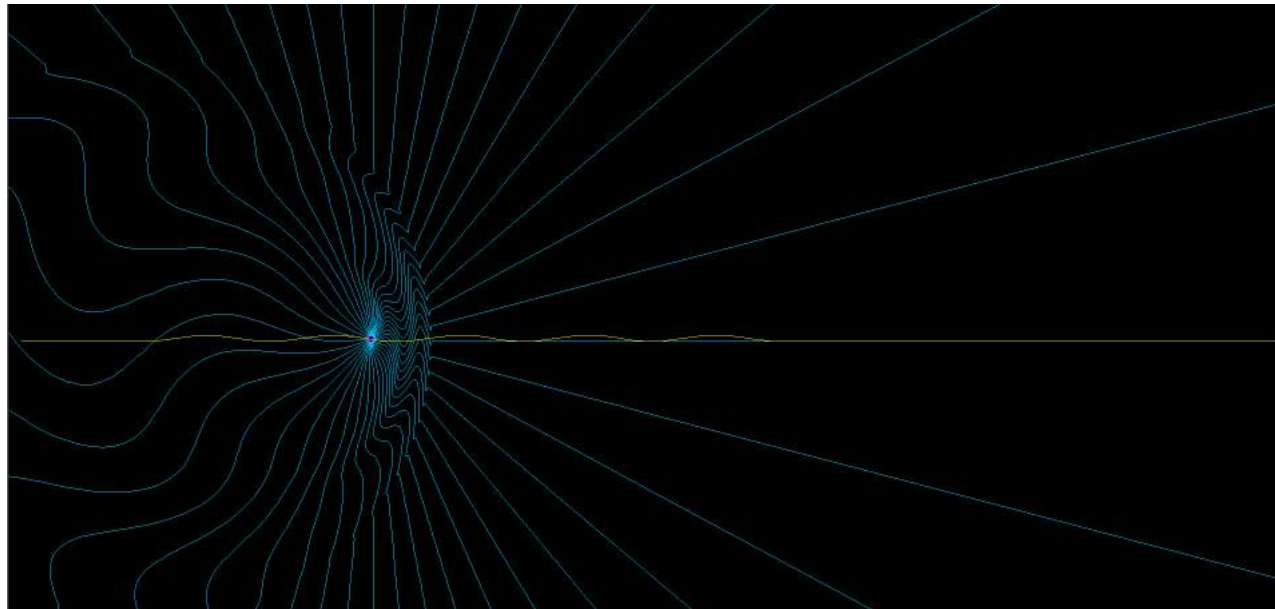




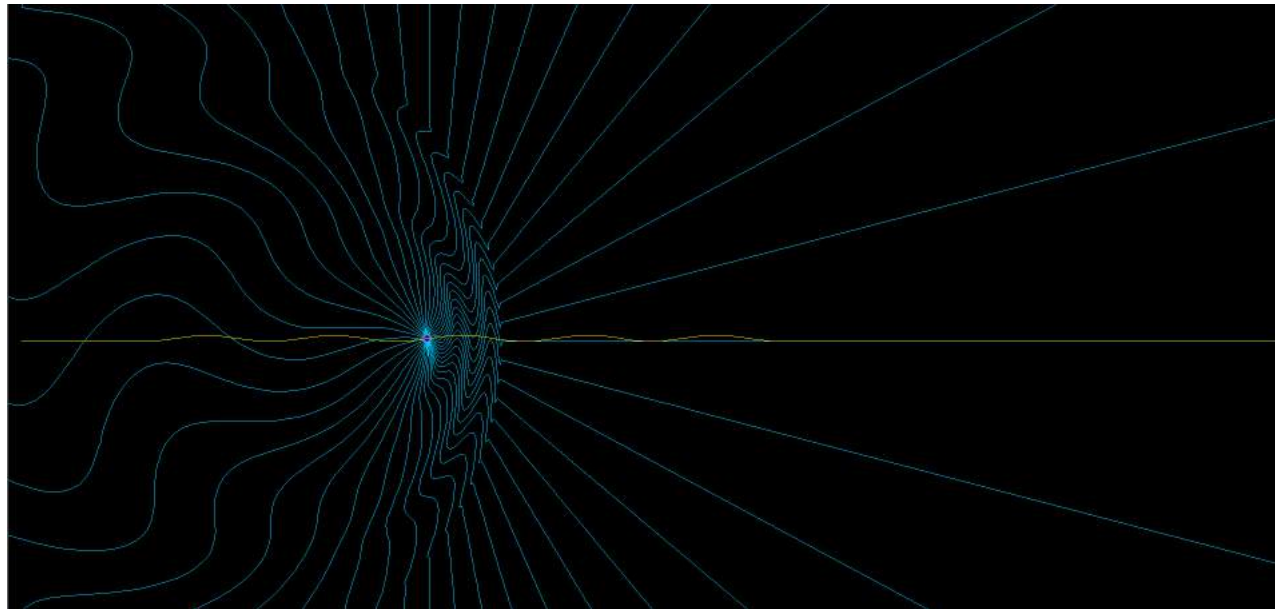
# RADIATION FROM ULTRARELATIVISTIC PARTICLES: UNDULATOR



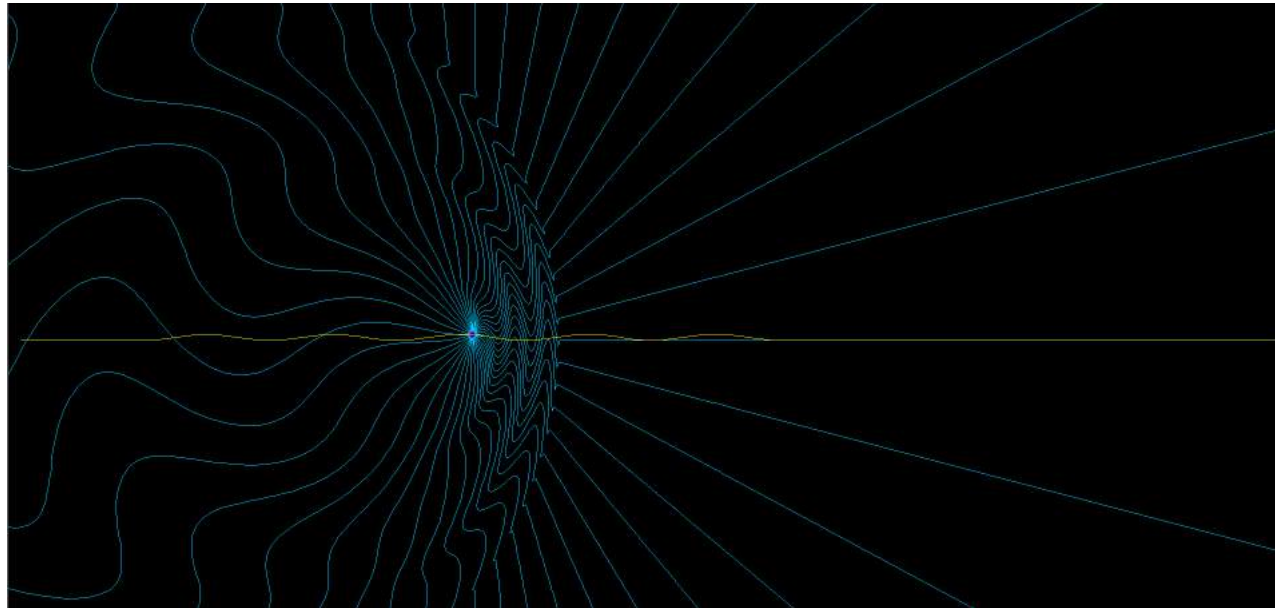
# RADIATION FROM ULTRARELATIVISTIC PARTICLES: UNDULATOR



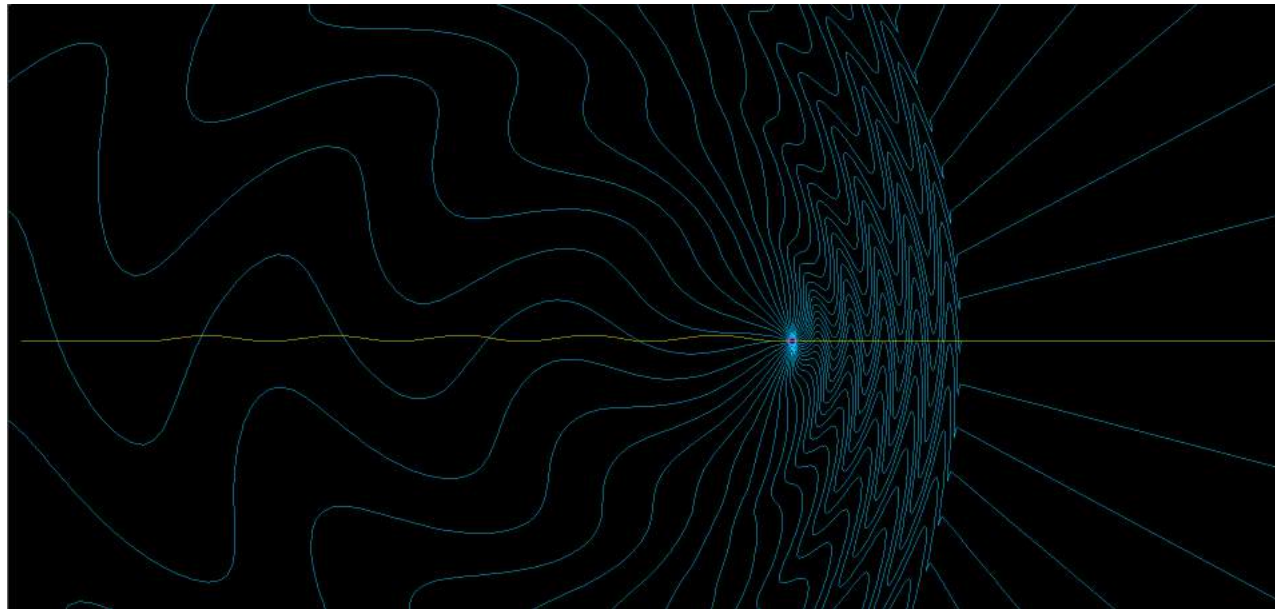
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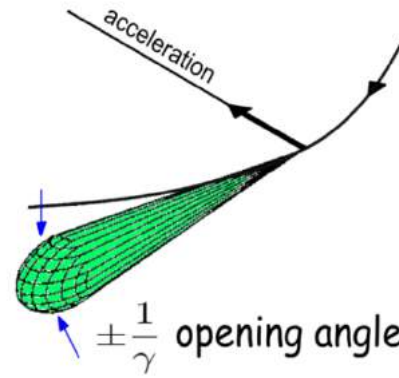
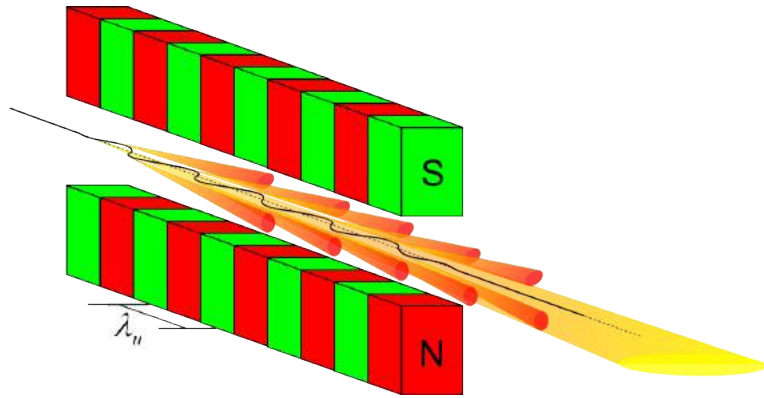


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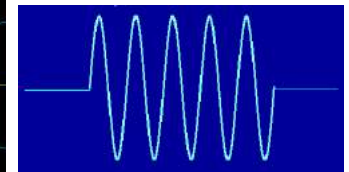
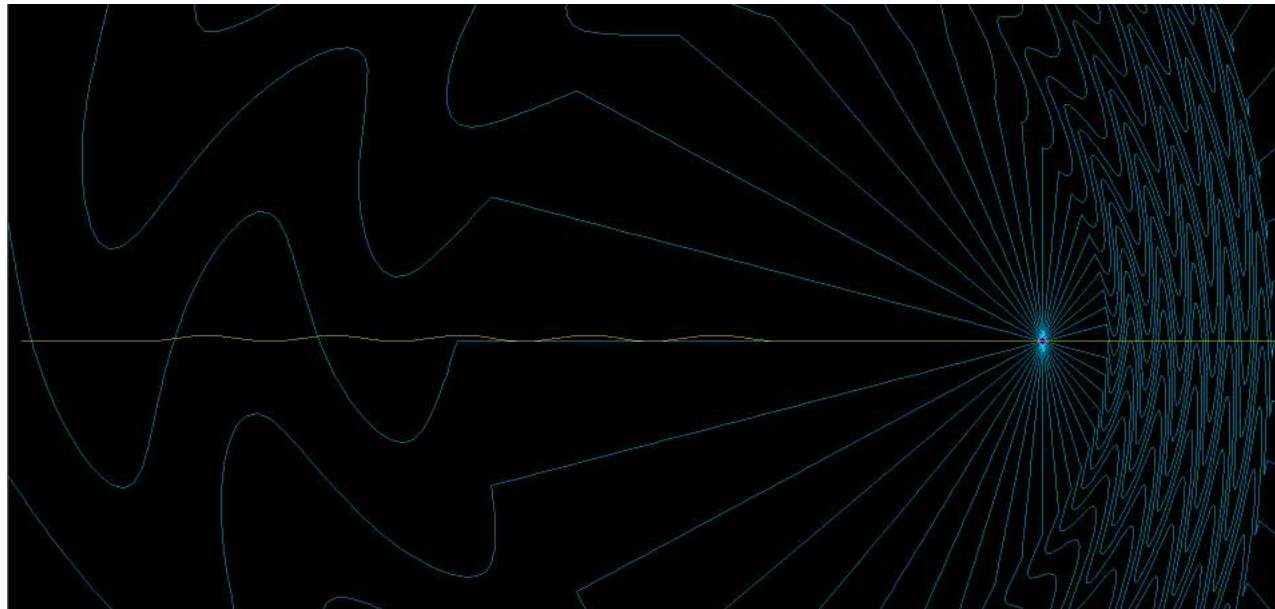




# RADIATION FROM ULTRARELATIVISTIC PARTICLES: UNDULATOR

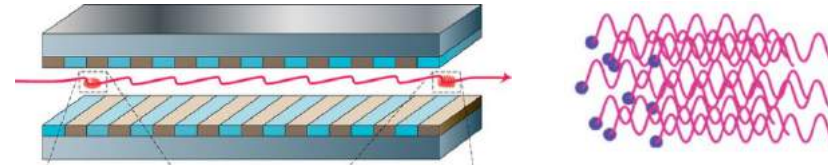
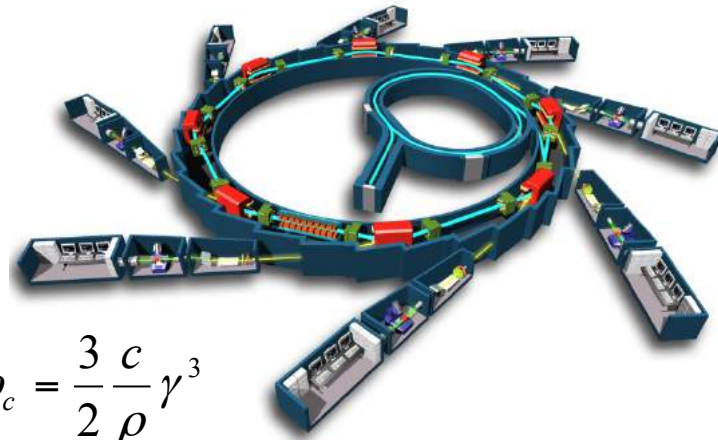


$$L_{pulse} = N_u \lambda_{rad}$$



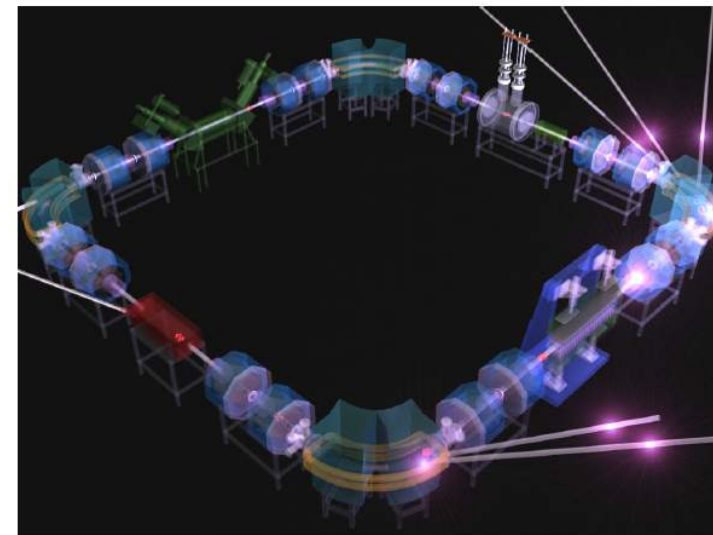
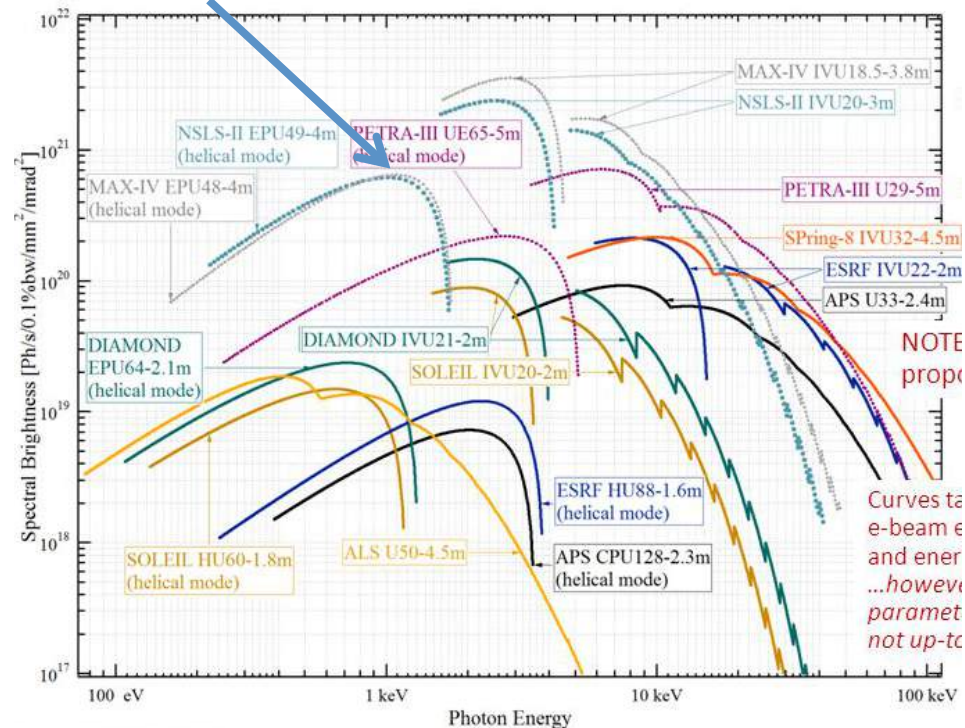
# PROPERTIES OF THE SYNCHROTRON RADIATION

Synchrotron radiation is emitted in a **broad spectrum**. The critical wavelength identifies the **peak of the spectrum** and is a function of the electron energy ( $\gamma^3$ ).



$$\omega_c = \frac{3}{2} \frac{c}{\rho} \gamma^3$$

$$P_{rad} = \frac{2}{3} \frac{cq^2}{4\pi\epsilon_0} \frac{\beta^4 \gamma^4}{\rho^2}$$

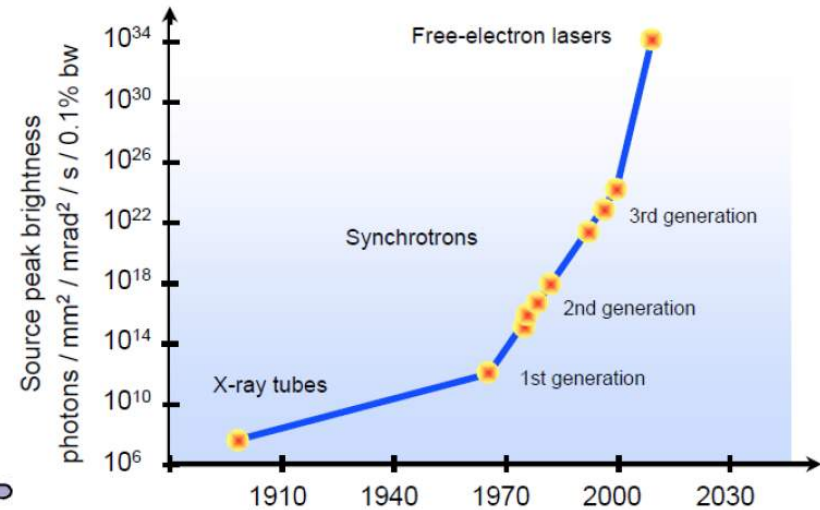
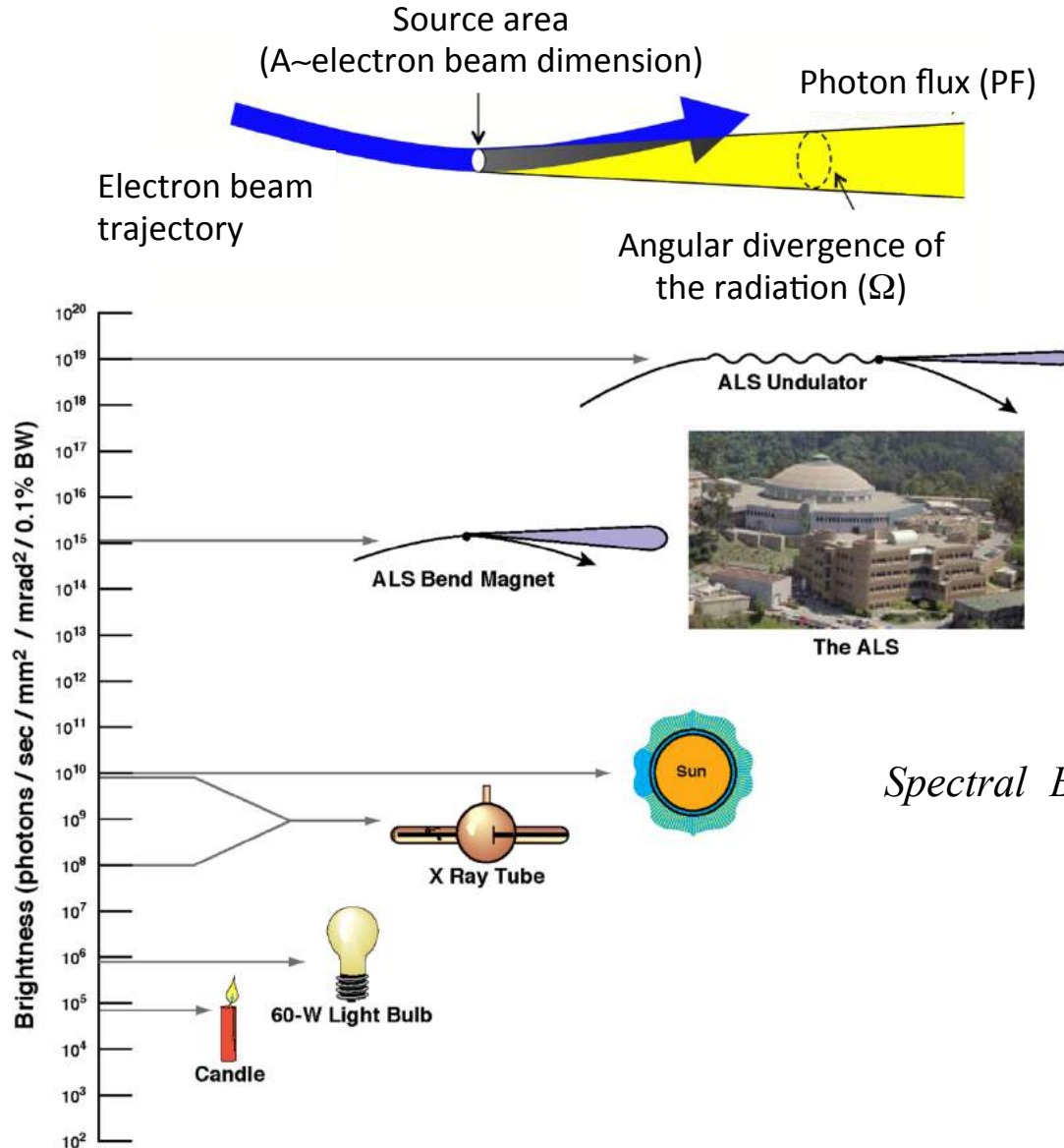




<http://www.isa.au.dk/animations/animations.asp>

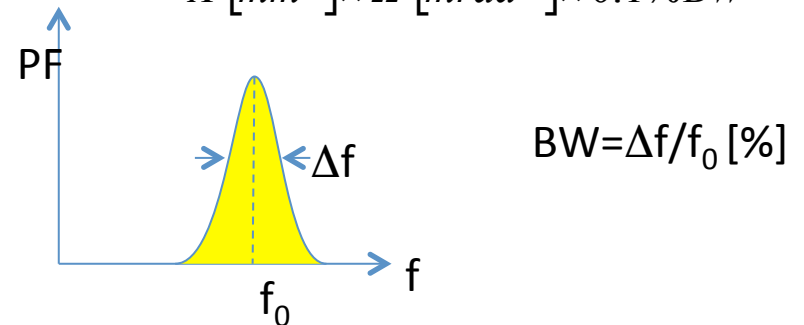
# BRIGHTNESS OF A PHOTON SOURCE

The most important parameter for a light source is not (only) the number of emitted photons per second but their “density” called **brightness**.



$$\text{Brightness} = \frac{PF [ph/s]}{A [mm^2] \times \Omega [mrad^2]}$$

$$\text{Spectral Brightness} = \frac{PF [ph/s]}{A [mm^2] \times \Omega [mrad^2] \times 0.1\% BW}$$





# EUROPEAN SYNCHROTRON SOURCES





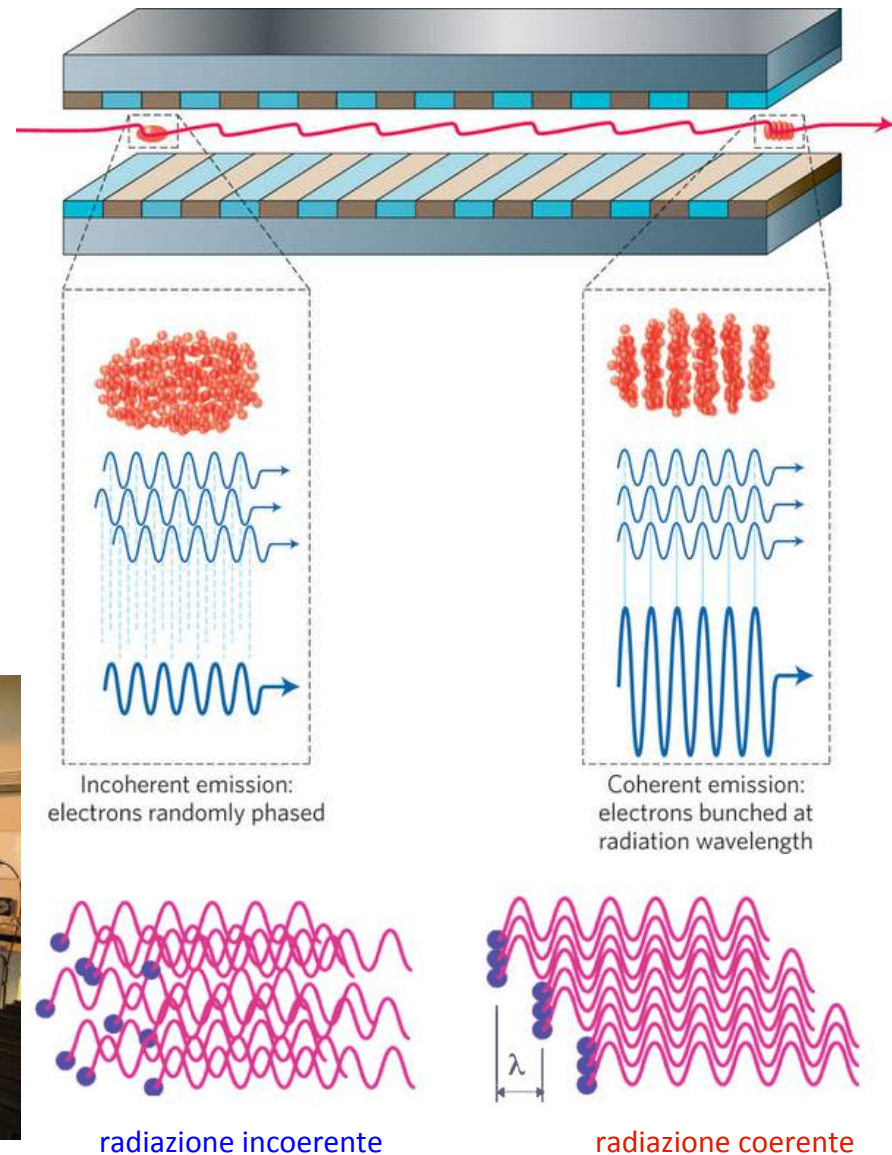
# Laser ad elettroni liberi: *Free Electron Lasers (FEL)*

I Laser ad Elettroni Liberi sono *potenti sorgenti di radiazione elettromagnetica coerente* (microonde, UV, raggi X) con *alta potenza di picco e alta brillantezza* (ordini di grandezza superiori agli anelli di luce di sincrotrone).

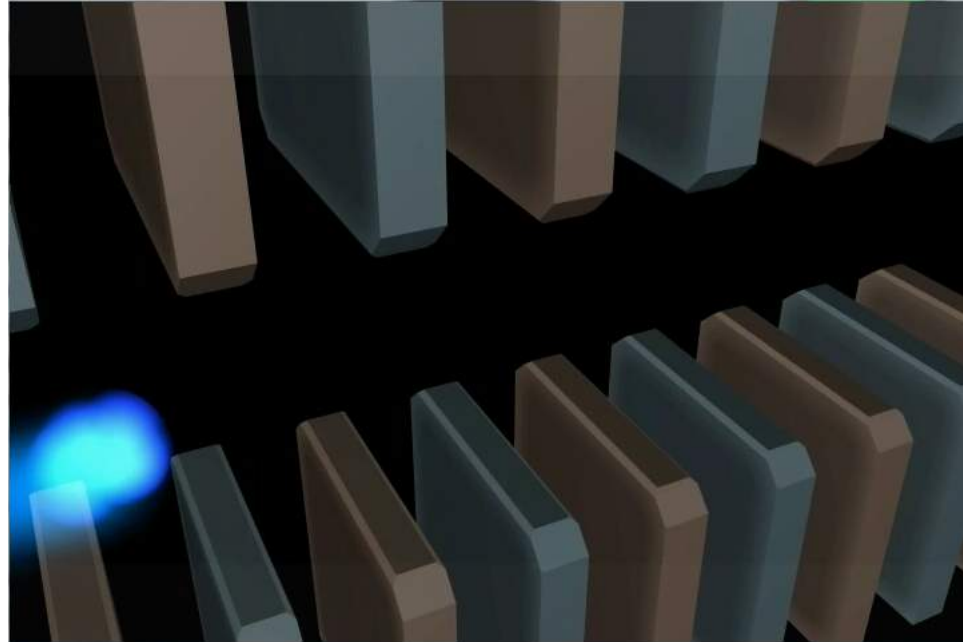
Un **LINAC ad e-** accelera pacchetti di elettroni di alta qualità (brillantezza) che, entrando nell'ondulatore, generano radiazione EM coerente, con un'amplificazione esponenziale.

All'interno degli ondulatori si ha in particolare una interazione luce emessa-elettroni del pacchetto che porta ad un fenomeno di **auto-impacchettamento (micro-bunching)** del pacchetto di elettroni su scala della lunghezza d'onda della radiazione emessa. I vari elettroni impacchettati emettono così coerentemente.

Tale tipo di radiazione ha enormi applicazioni poiché consente analisi di strutture anche non cristallizzate.



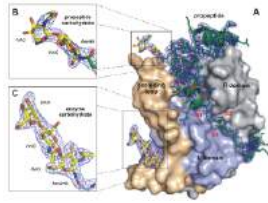
# FEL: video



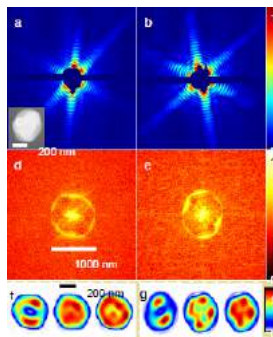
# FEL: RADIAZIONE COERENTE ED IMPULSI ULTRA-CORTI

⇒ la **radiazione coerente** emessa da un FEL consente di «fotografare» anche molecole o sistemi non cristallizzati

⇒ con i FEL è possibile generare **impulsi ultra-corti** (fs) con cui è possibile «filmare» movimenti di molecole, passaggi di carica etc...

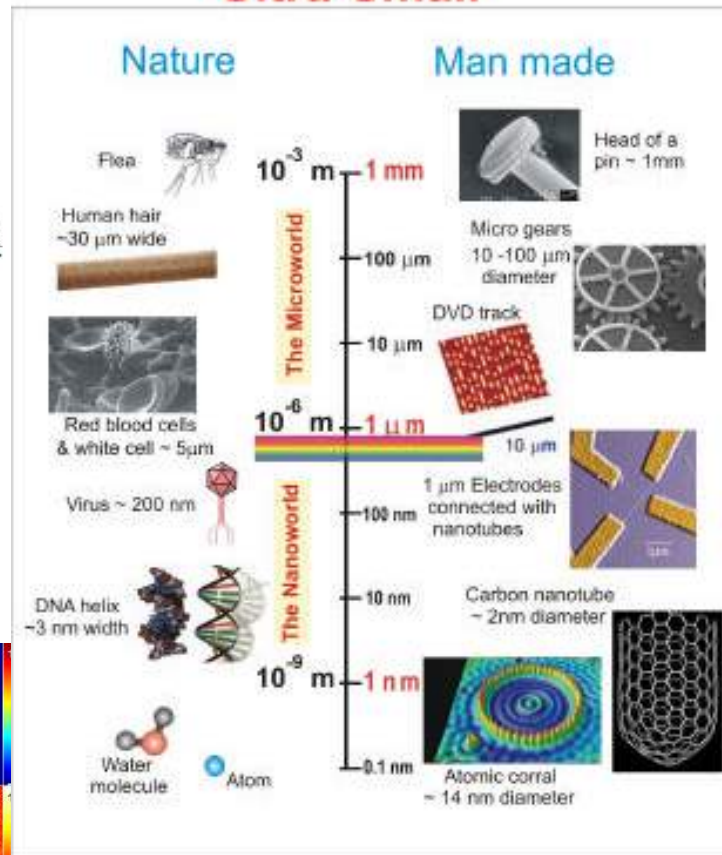


Inhibited Trypanosoma brucei Cathepsin B Structure Determined by Using an Xray Laser, L. Redecke et al. Science 339, 227 (2013)



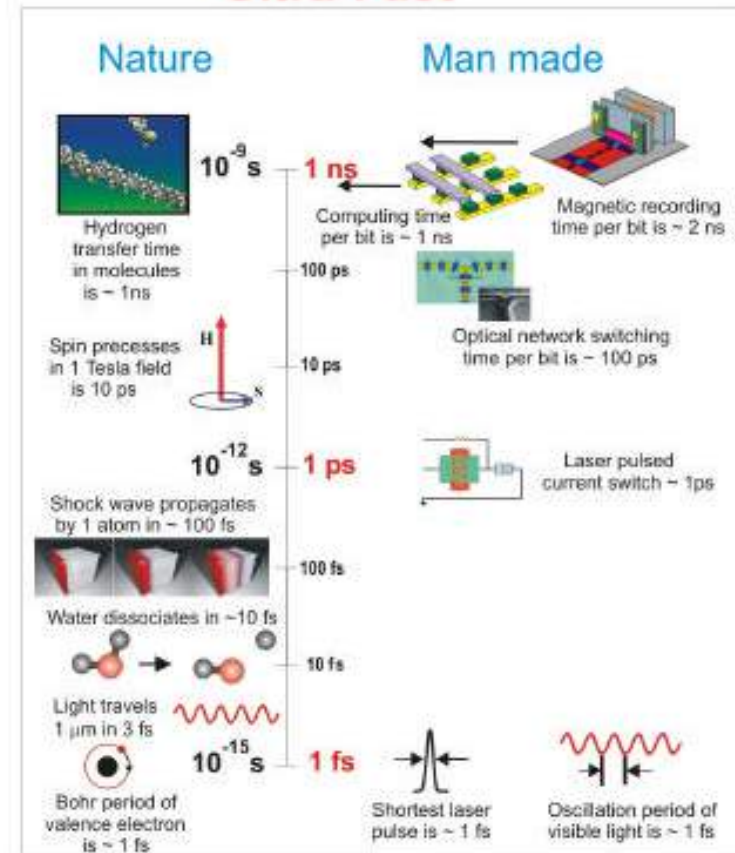
Imaging single mimivirus Seibert et al, Nature, 470, 78 (2011)

## Ultra-Small

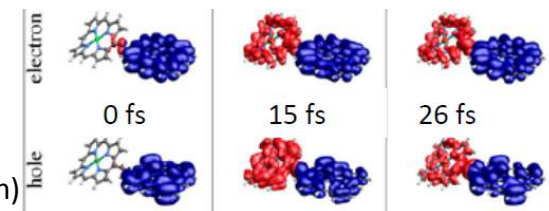


Courtesy C. Pellegrini and J. Stohr

## Ultra-Fast

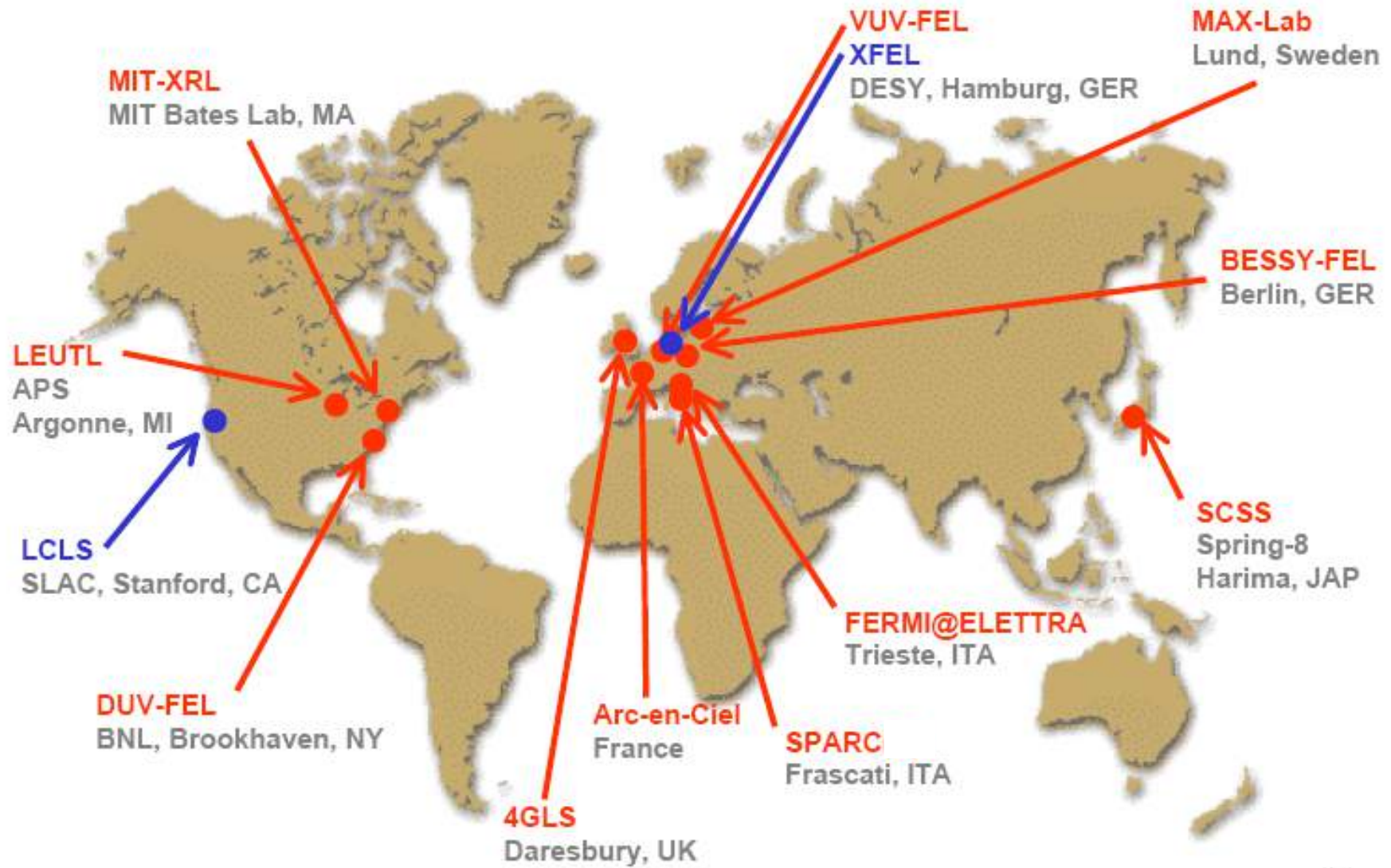


Simulated X-ray Raman Redistribu'on: (example: Metalloporphyn)





# FEL nel mondo

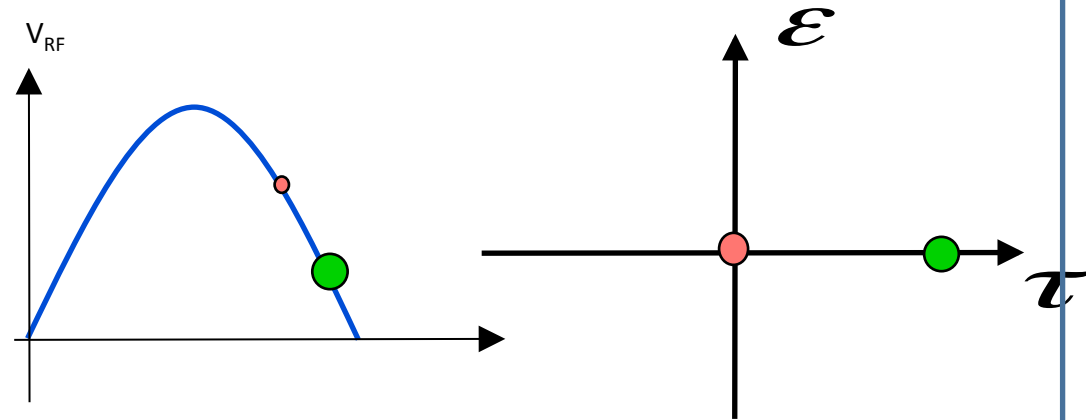


# GAUSSIAN DISTRIBUTION OF ELECTRON BUNCHES IN ELECTRON RINGS

The **synchrotron light emission** in electron (or positron) rings leads to a series of **consequences**:

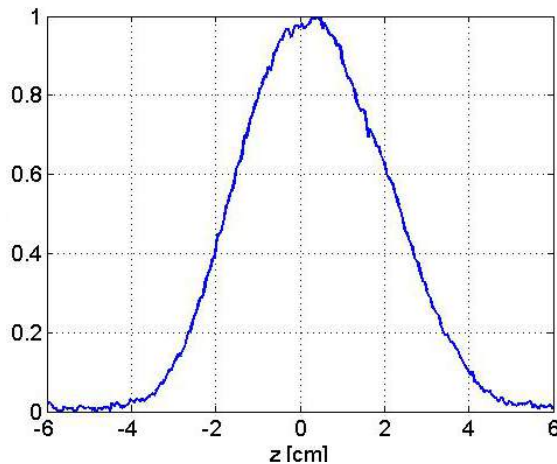
→ The particles lose energy at each turn and this energy must be supplied to the beam by the **accelerating cavity** to avoid that the particles, becoming less energetic, are lost.

→ Since the emission depends on the energy of the particle, there is a **damping effect** on the amplitude of the synchrotron oscillation (radiation damping) which would tend to bring all the particles on the synchronous phase (damped oscillator system).



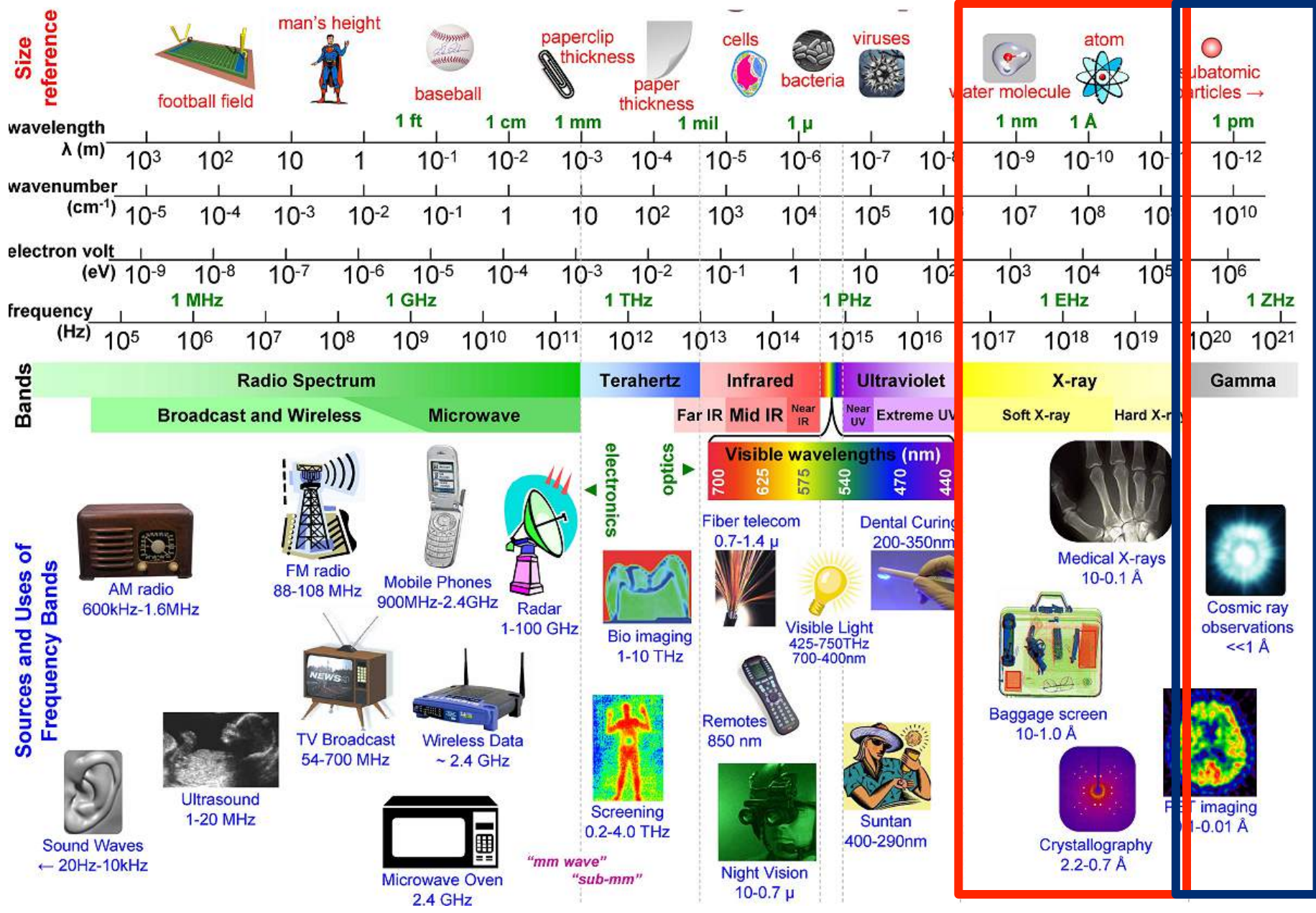
→ On the other hand, since radiation is emitted in the form of quanta of light, it generates a "noise" (**quantum excitation**) that would tend to increase the amplitude of the oscillation.

At the end we arrive at a situation of equilibrium between these two opposite phenomena which tends to make the electrons bunches with a **gaussian profile**

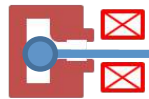




# PHOTONS WITH DIFFERENT WAVELENGTH: $\gamma$ -RAYS



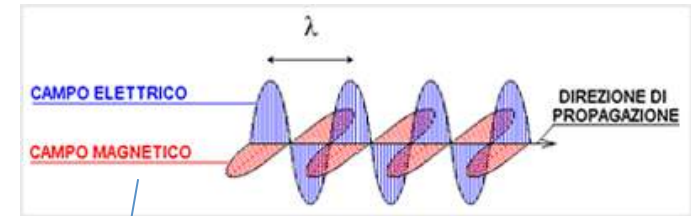
# SOURCE OF $\gamma$ -RAYS BASED ON ELECTRONIC-PHOTON COLLISIONS



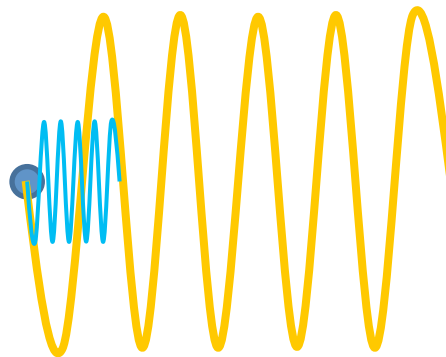
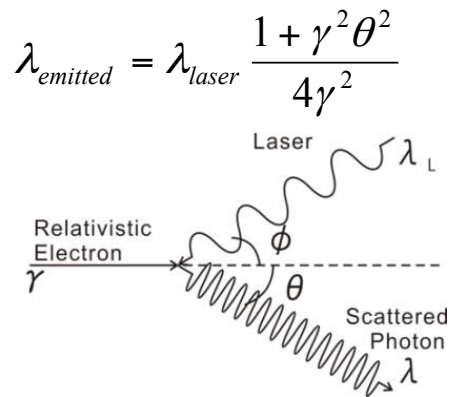
Electron source



LINAC (100-800 MeV)



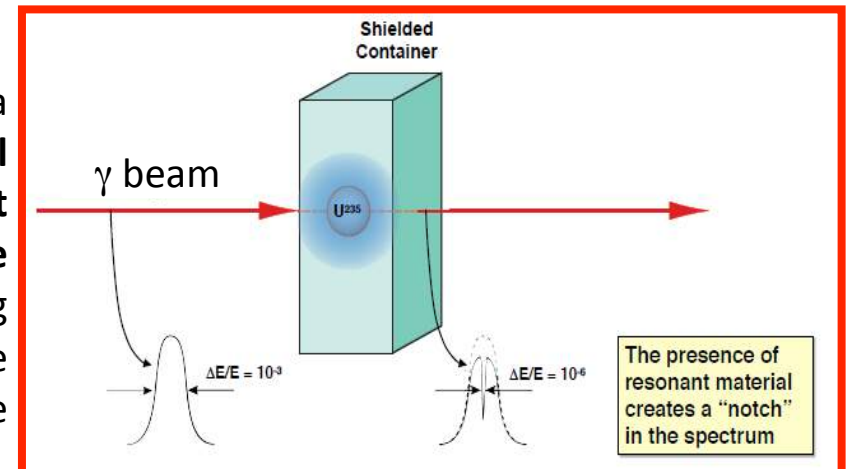
LASER



The emitted photon beam can cover a radiation spectrum from X-rays to  $\gamma$ -rays



The  $\gamma$ -photons have a wavelength so small that they can interact with the nuclei of the atoms giving information on the nuclear structure of the atom itself



The presence of resonant material creates a "notch" in the spectrum



# ACCELERATOR BASED NEUTRON SOURCES

**The Nobel Prize in Physics 1994**

*Neutrons behave like particles and waves*

The Royal Swedish Academy of Sciences has awarded the 1994 Nobel Prize in Physics for pioneering contributions to the development of neutron scattering techniques for studies of condensed matter.

**Clifford G. Shull, 1915**, **Clinton Davisson, 1915**, and **George Paget Thomson, 1927**, were awarded the 1937 Nobel Prize in Physics for their development of the neutron diffraction technique.

**Clifford G. Shull, 1915**, **Clinton Davisson, 1915**, and **George Paget Thomson, 1927**, were awarded the 1937 Nobel Prize in Physics for their development of the neutron diffraction technique.

## Neutrons reveal structure and dynamics

**S** Shull made use of atoms scattering low energy neutrons which change direction without losing energy when they collide with atoms.

**B** Thomson made use of ballistic scattering, i.e. of neutrons which change both direction and energy when they collide with atoms. They then gave us crystal structure information by crystals and neutron instruments in liquid and solids. Neutrons can also interact with spin waves in magnets.

With the SNS experimenters' breakthrough measured energies of phonons (atomic vibrations) and magnons (magnetic waves) they also studied how atomic structures in liquids change with time.

Neutrons show where atoms are

Neutrons source against atomic nuclei. They also react to the magnetism of the atoms.

Neutrons show what atoms do

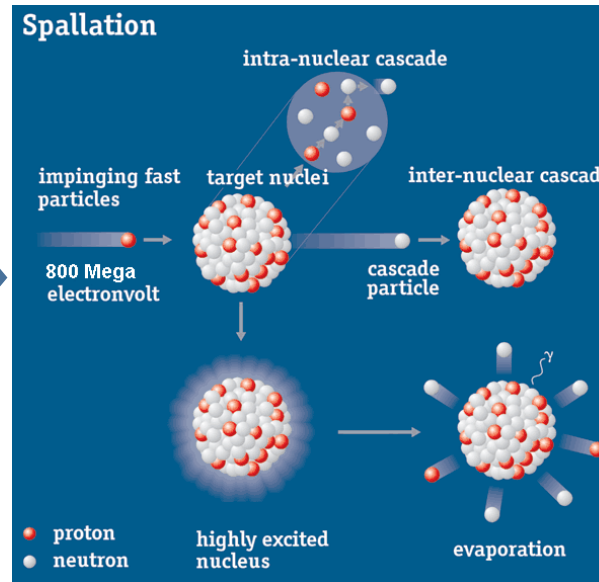
When the neutron penetrates the atoms in the crystal, it is scattered by the nuclei. The scattered neutrons are then detected by a detector.

Neutrons are more than X-rays

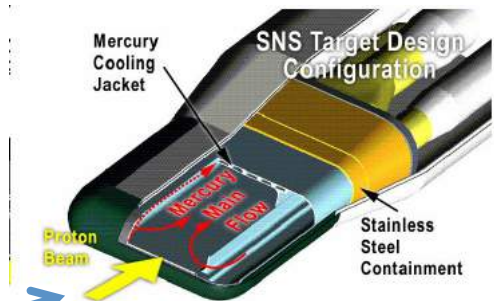
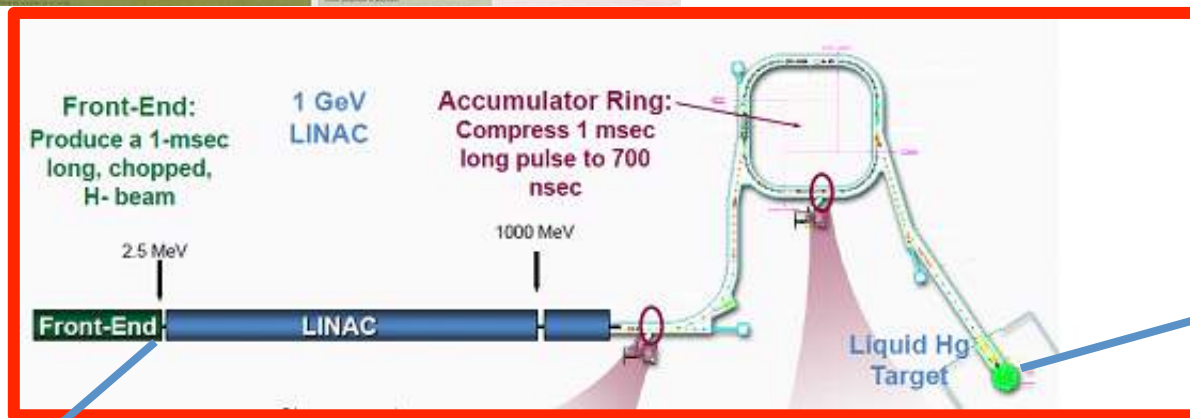
Neutrons reveal lower stresses

Neutrons show what stress is

Further reading



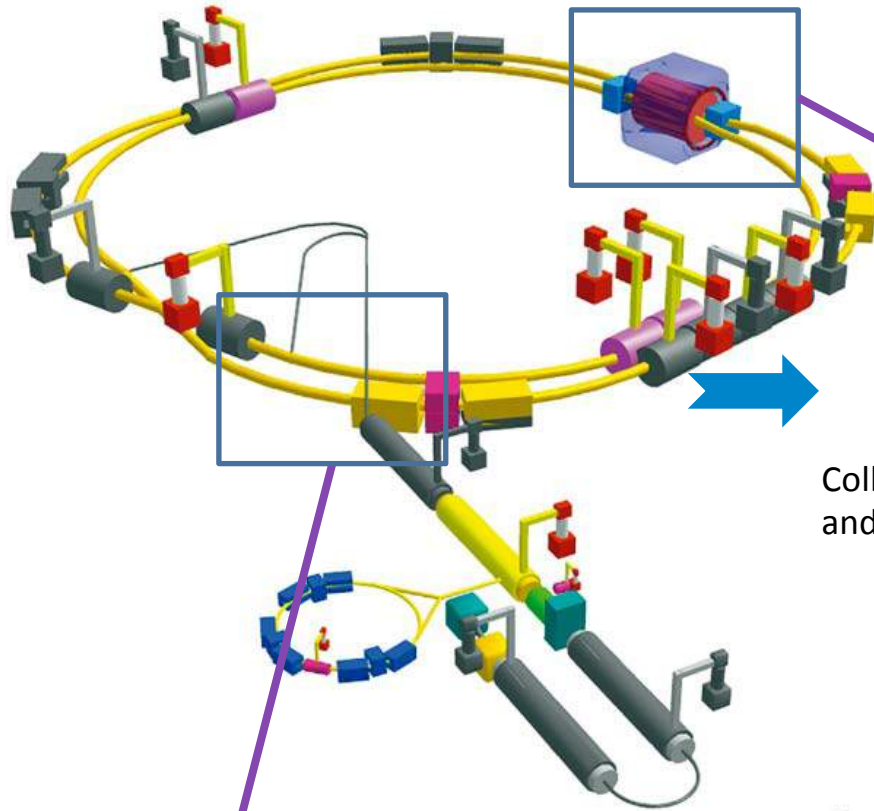
spallation



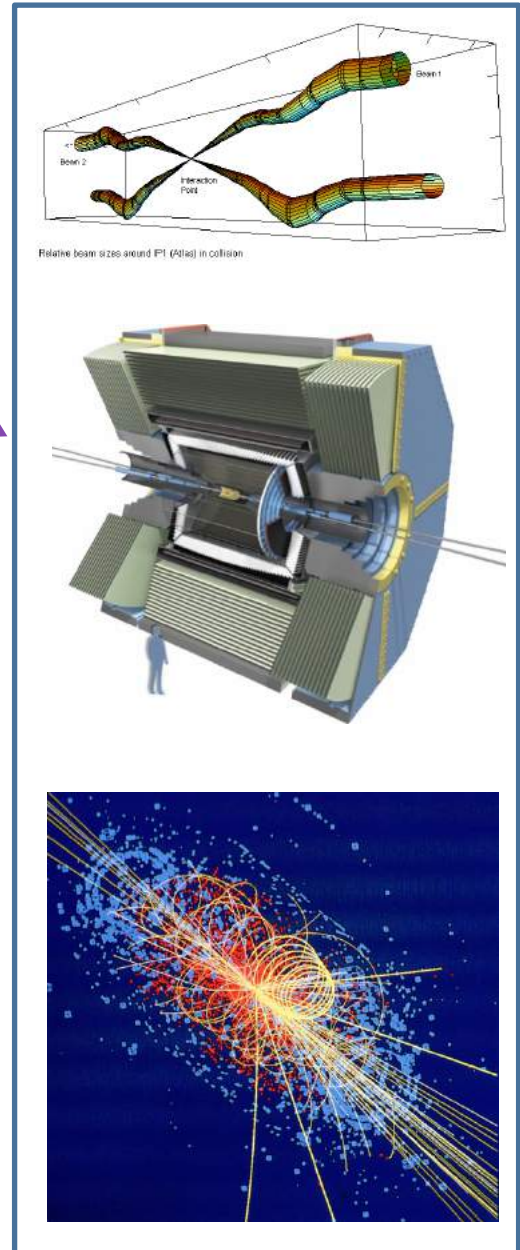
$10^{15} \text{ n/s/cm}^2$



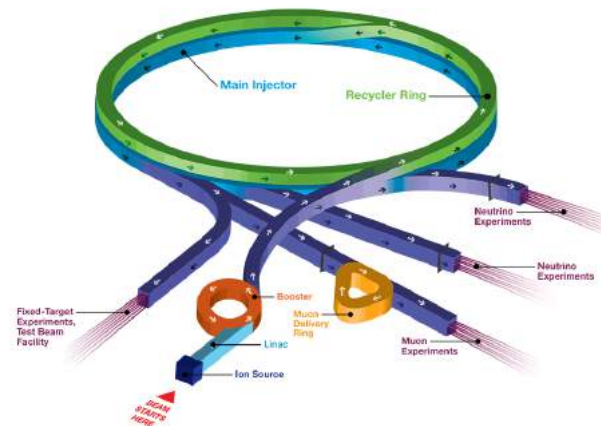
# FUNDAMENTAL PHYSICS: COLLIDERS



Collision between particles and targets



Fermilab Accelerator Complex



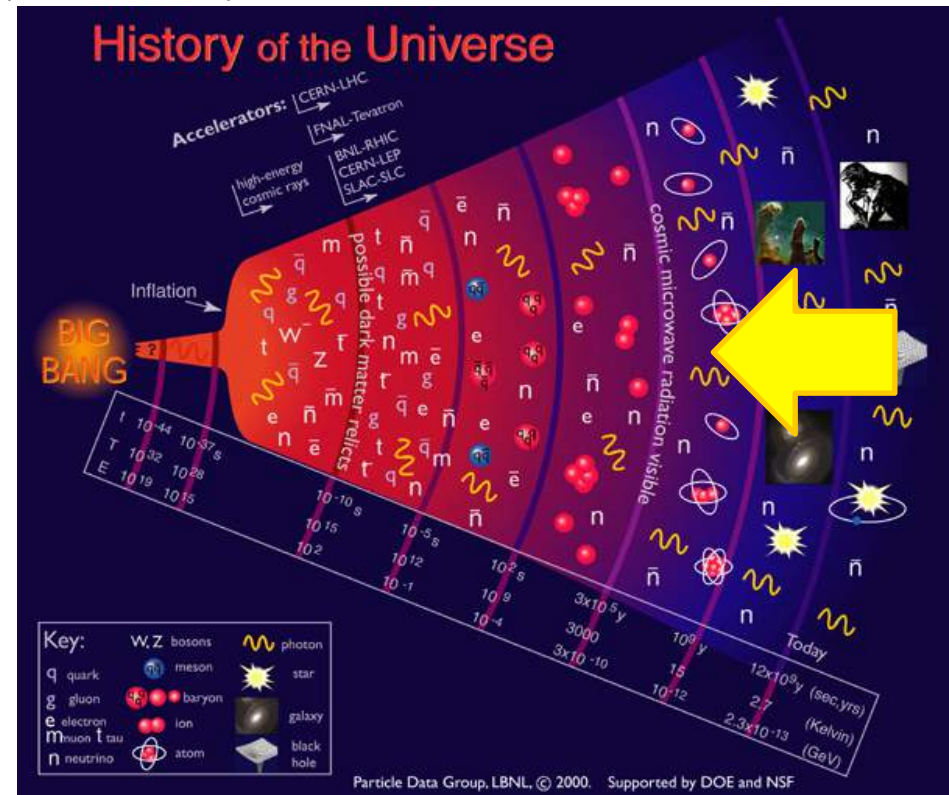
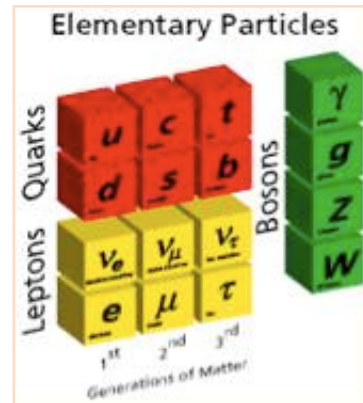
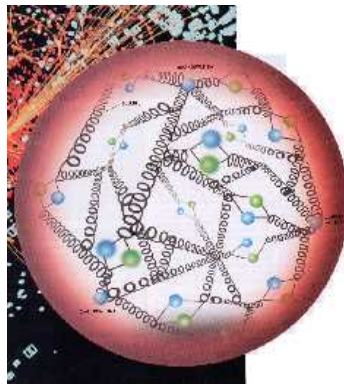


# PARTICLE ACCELERATORS: SUB-ATOMIC MICROSCOPES AND TIME MACHINES

The **collisions** between two beams of particles or between a beam of particles and a target...

provide information on the ultimate “brick” of our universe and on their laws (**microscope**)

Recreating higher and higher energy densities, allow to go back to the first moments of Universe life and to study its evolution (**time machine**)



# COLLISIONS: ENERGY OR ENERGY DENSITY?

The eV represents a very small amount of energy

$$1 \text{ eV} = 1\text{V} \times 1.6 \times 10^{-19} \text{ C} = 1.6 \times 10^{-19} \text{ J}$$

$$1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$$

$$1 \text{ GeV} = 1.6 \times 10^{-10} \text{ J}$$

A 600 g iron bullet shot at 300 m/s has an energy of 27000 J



But every single proton or neutron of the bullet has a very small kinetic energy  $27000/N_{p+n} \approx 8 \cdot 10^{-22} \text{ J} \approx \mathbf{0.005 \text{ eV}}$ !



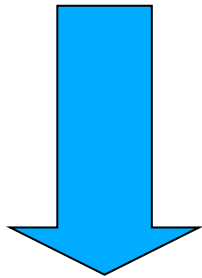
In an accelerator such as LHC for each proton we reach an energy up to 7 TeV ( $7 \times 10^{12} \text{ eV}$ )!

The **energy density available** for nuclear or sub-nuclear reactions is enormous!

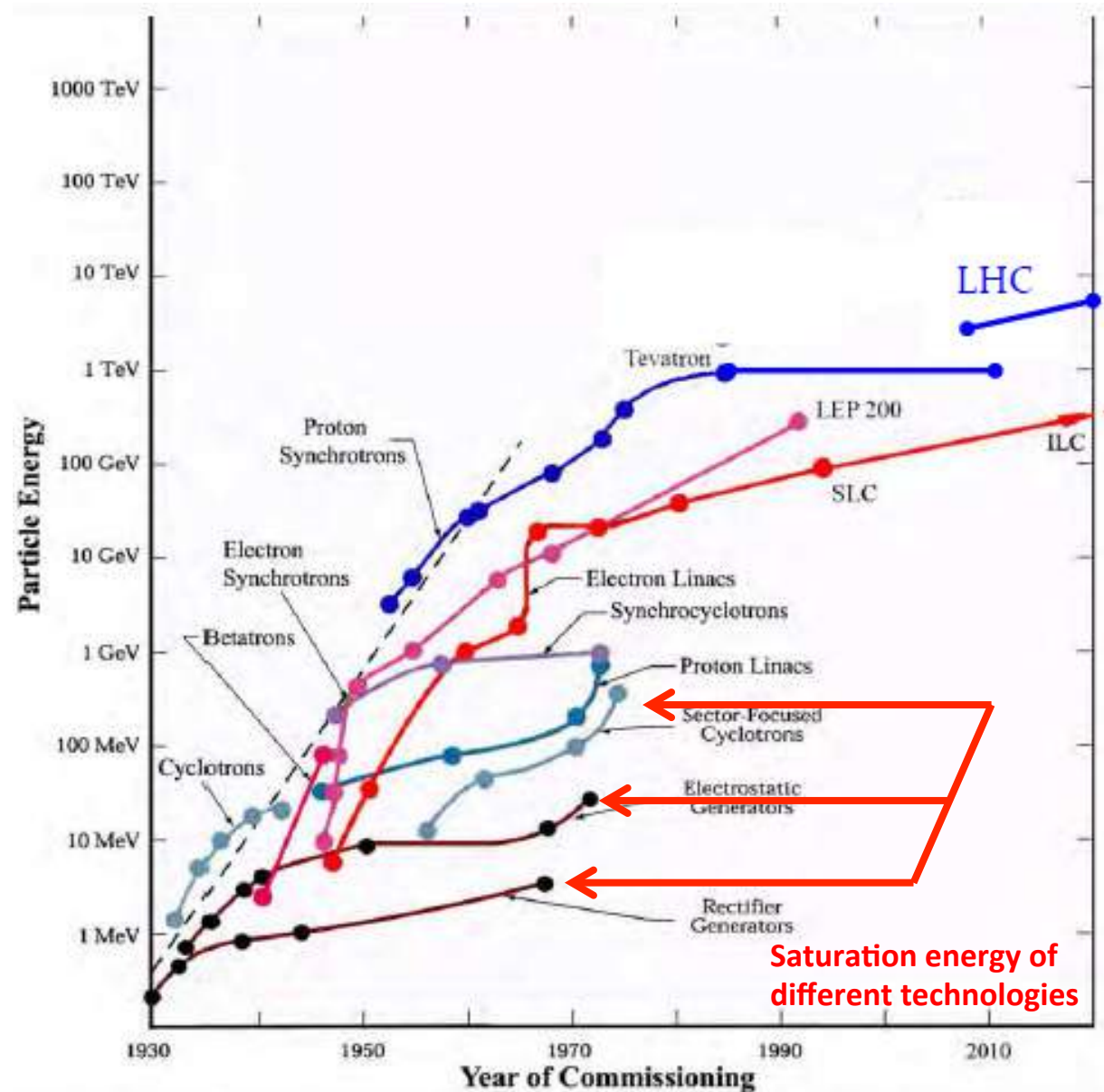


# DEVELOPMENT OF PARTICLE ACCELERATORS

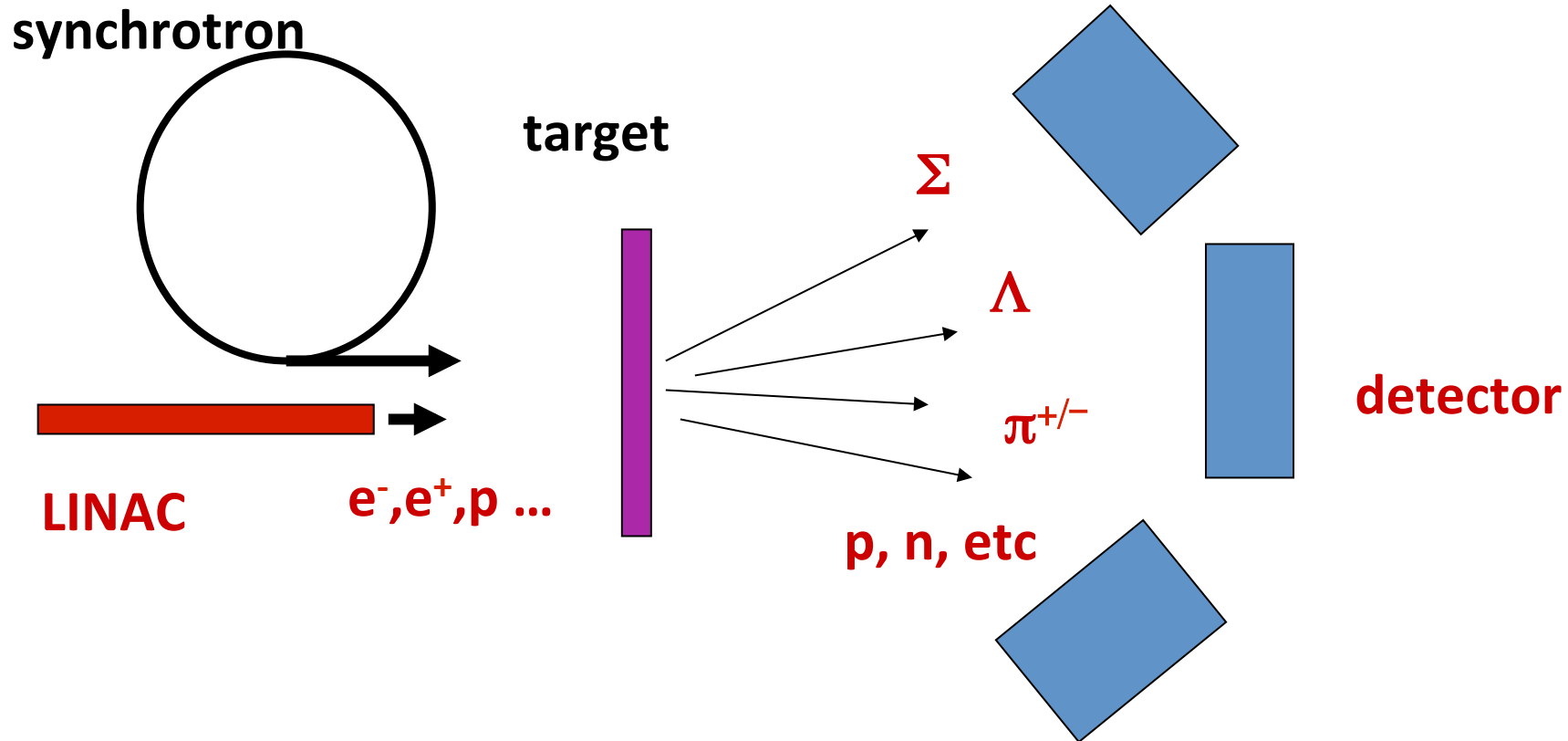
The ability to "create" new particles and "break" the nuclear structure increases with energy and with the amount of particles involved in the interaction.



The development of accelerators for high-energy physics has been determined by the need to obtain **energy and intensities** larger and larger.



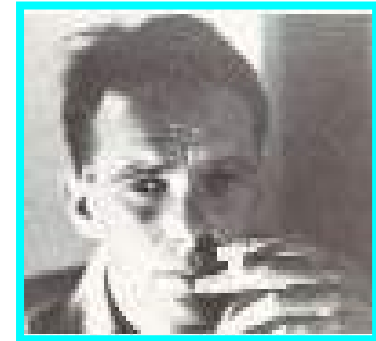
# PARTICLE PHYSICS WITH ACCELERATORS: COLLISIONS ON TARGETS



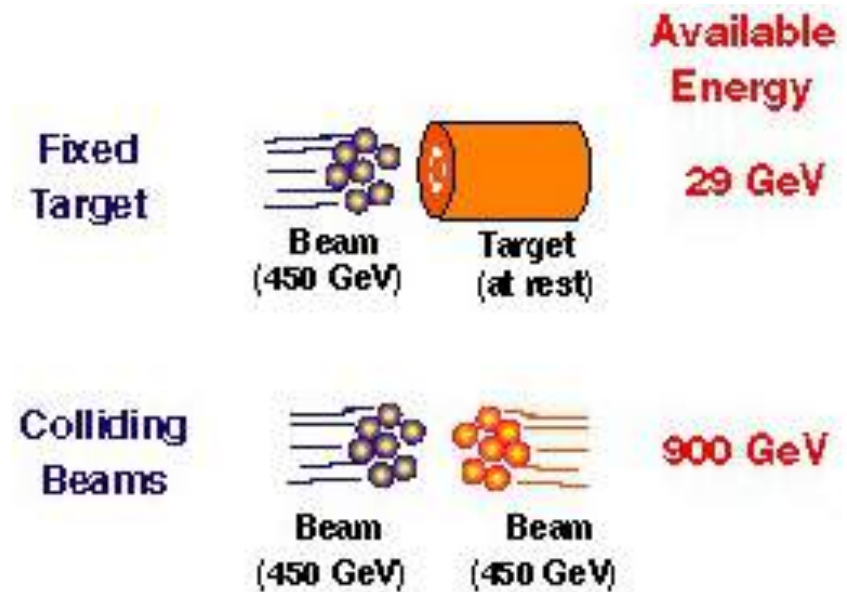
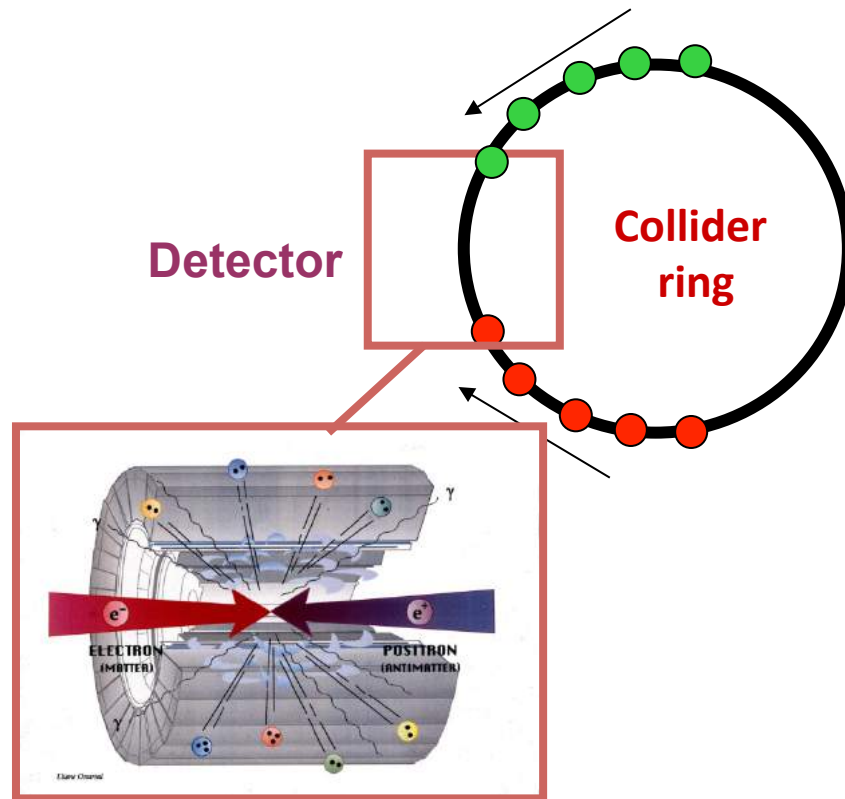
- 1) Matter is "**empty**": what has not interacted is lost;
- 2) The target is **complex**: many of the particles produced are only background for the experiment;
- 3) The detector is "**Anisotropic**" (one side);
- 4) Part of the energy of the particle beam is **not "used"** in the interaction and remains in the Center of Mass of the moving system;



# BIRTH OF THE MODERN COLLIDERS: TOUSCHEK

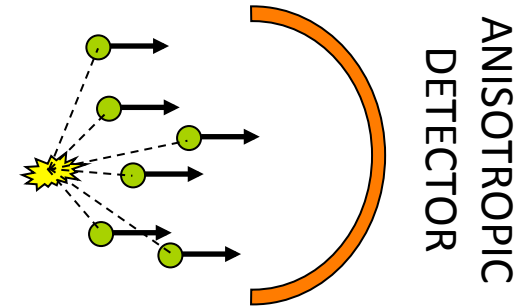
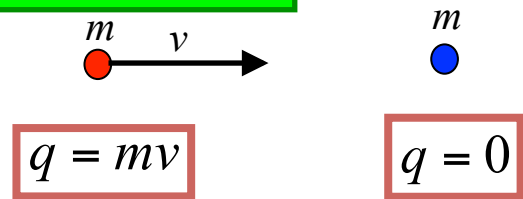


The brilliant idea of **Bruno Touschek (1960)** was to use colliding **particles and antiparticles** that, in their annihilation, would release **all their energy** to create new particles. Furthermore, the collision products would have been relatively "**simple**" compared to those produced by collisions against a complex target.



# AVAILABLE ENERGY IN THE INTERACTION

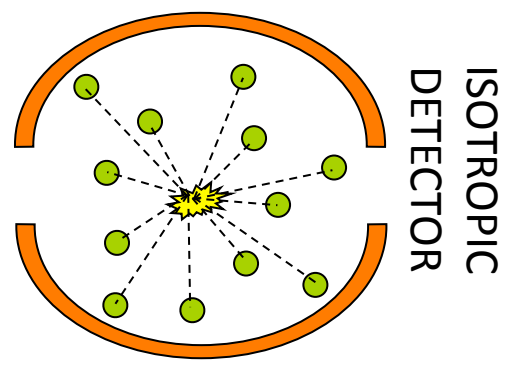
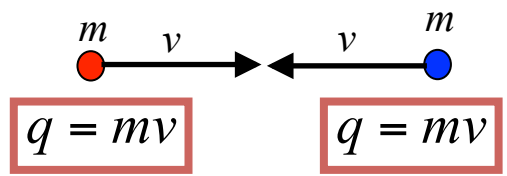
## COLLISION AGAIN A FIXED TARGET



Available energy in the interaction

$$\sqrt{2Emc^2}$$

## HEAD-ON COLLISION

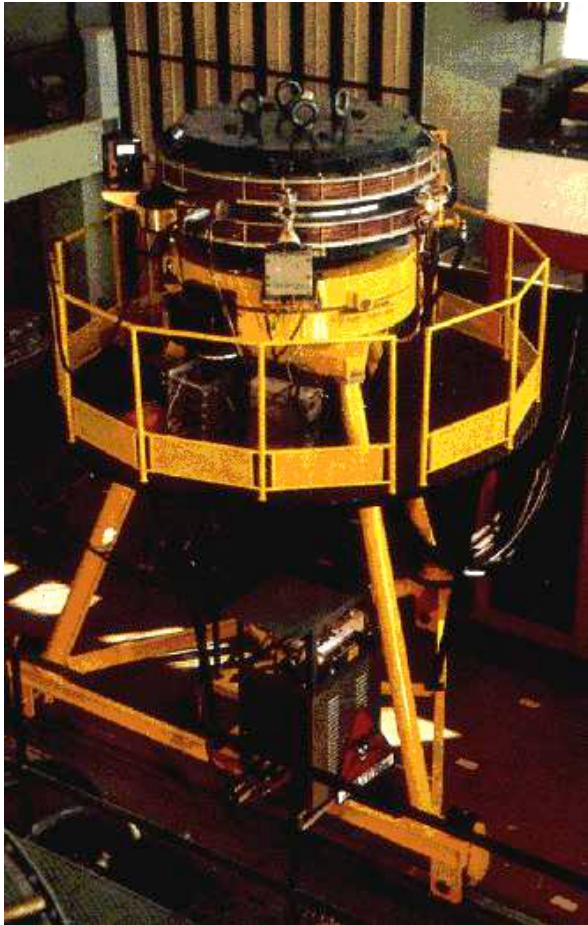


Available energy in the interaction

$$E_{CM} \approx 2E$$

# AdA (Anello di Accumulazione) 1960-1965

AdA is made of a weak focusing magnet able to circulate particles (e<sup>+</sup>/e<sup>-</sup>) with an energy of 250 MeV.



Length of orbit . . . . .	408	cm
Radio frequency ( $k=2$ )	147	MHz
Radio voltage . . . . .	10	kV
Length of bunches . . . . .	16.7	cm
Radial width of bunches	.22	cm
Height of bunches (at $10^{-10}$ mm) . . . . .	$5.6 \cdot 10^{-4}$	cm
Radiation loss/revolution	580	eV
Lifetime of beam at $10^{-10}$ mm . . . . .	250	h

IL NUOVO CIMENTO

The Frascati Storage Ring.

C. BERNARDINI, G. F. CORAZZA, G. GHIGO  
*Laboratori Nazionali del CNEN - Frascati*

R. TUSCHKEK

*Istituto di Fisica dell'Università - Roma*  
*Istituto Nazionale di Fisica Nucleare - Sezione di Roma*

(ricevuto il 7 Novembre 1960)

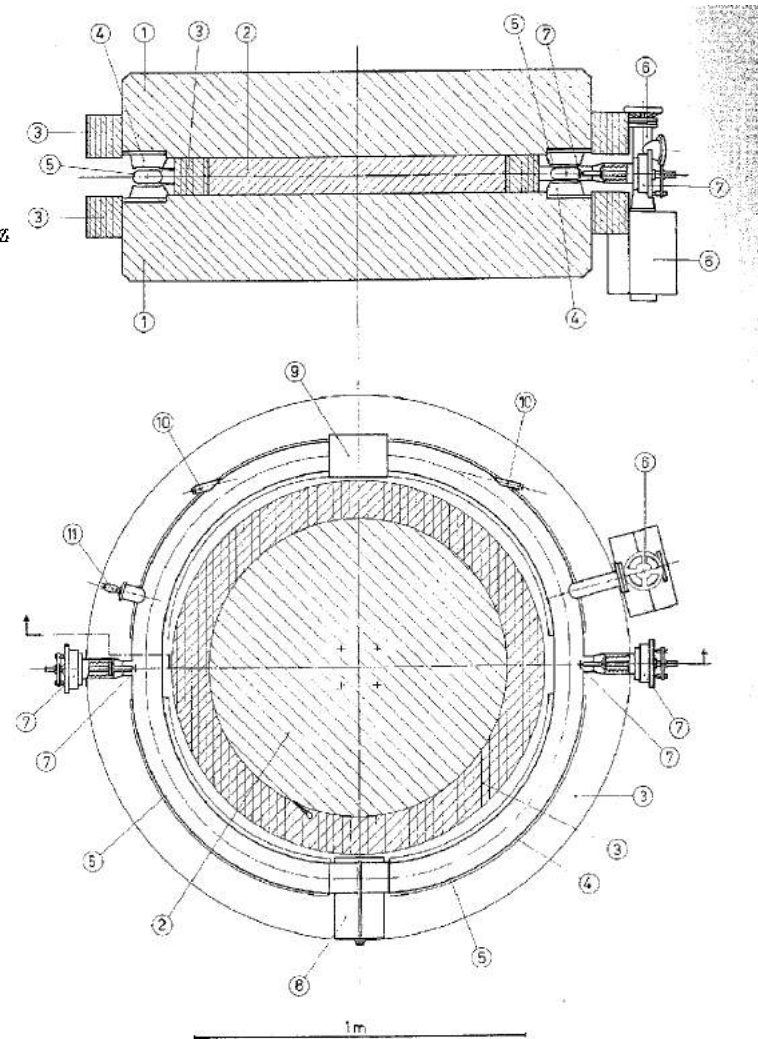
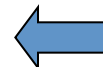
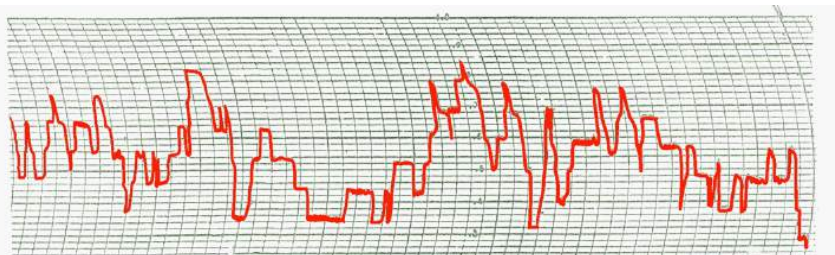


Fig. 1. - Elevation and plan section of the Frascati Storage Ring (anello di accumulazione - AdA): 1) magnet yoke; 2) magnet core; 3) coils; 4) polepieces; 5) doughnut; 6) titanium pump; 7) injection ports; 8) RF cavity; 9) experimental section; 10) windows for the observation of the synchrotron radiation; 11) vacuum gauge.

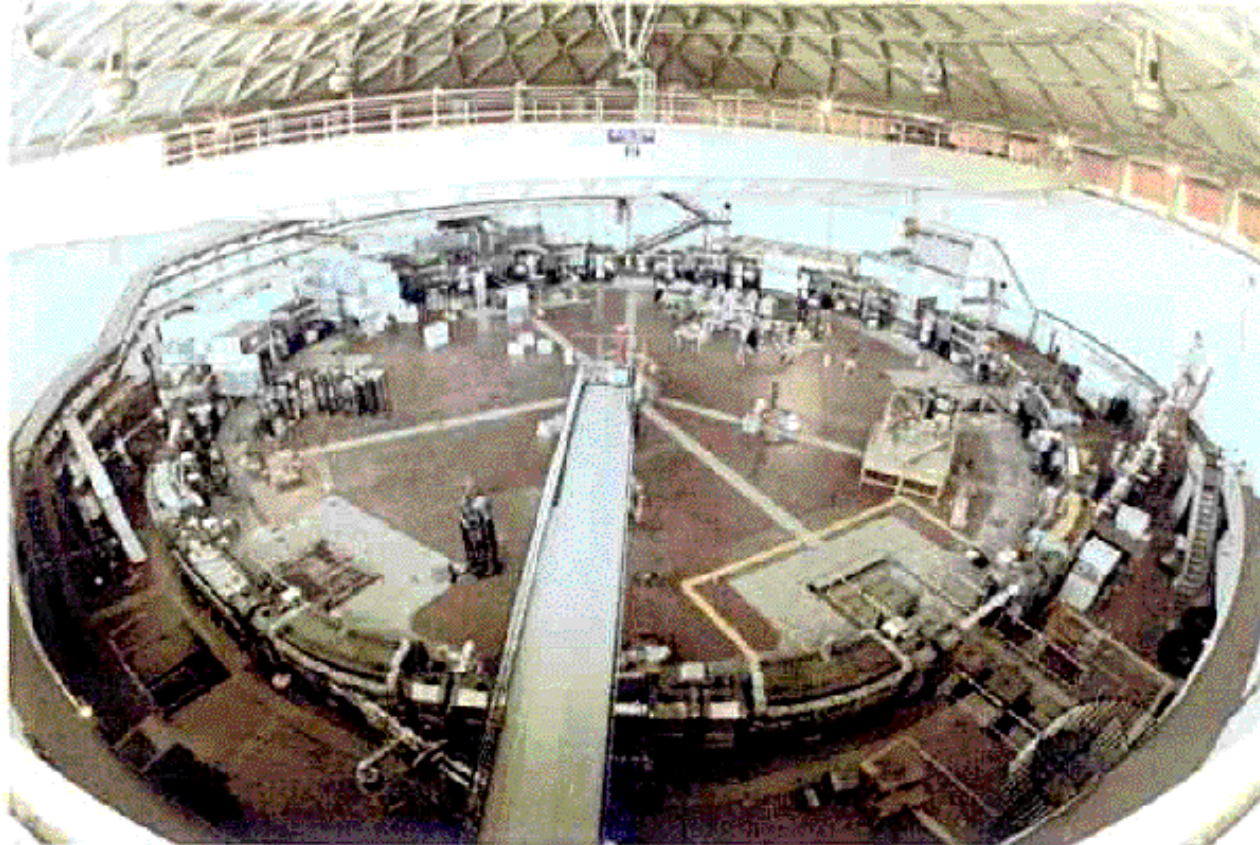


Trace of the first electrons accumulated in AdA. The average life was 21 sec, the average number 2.3.



# ADONE ( 1967-1993 )

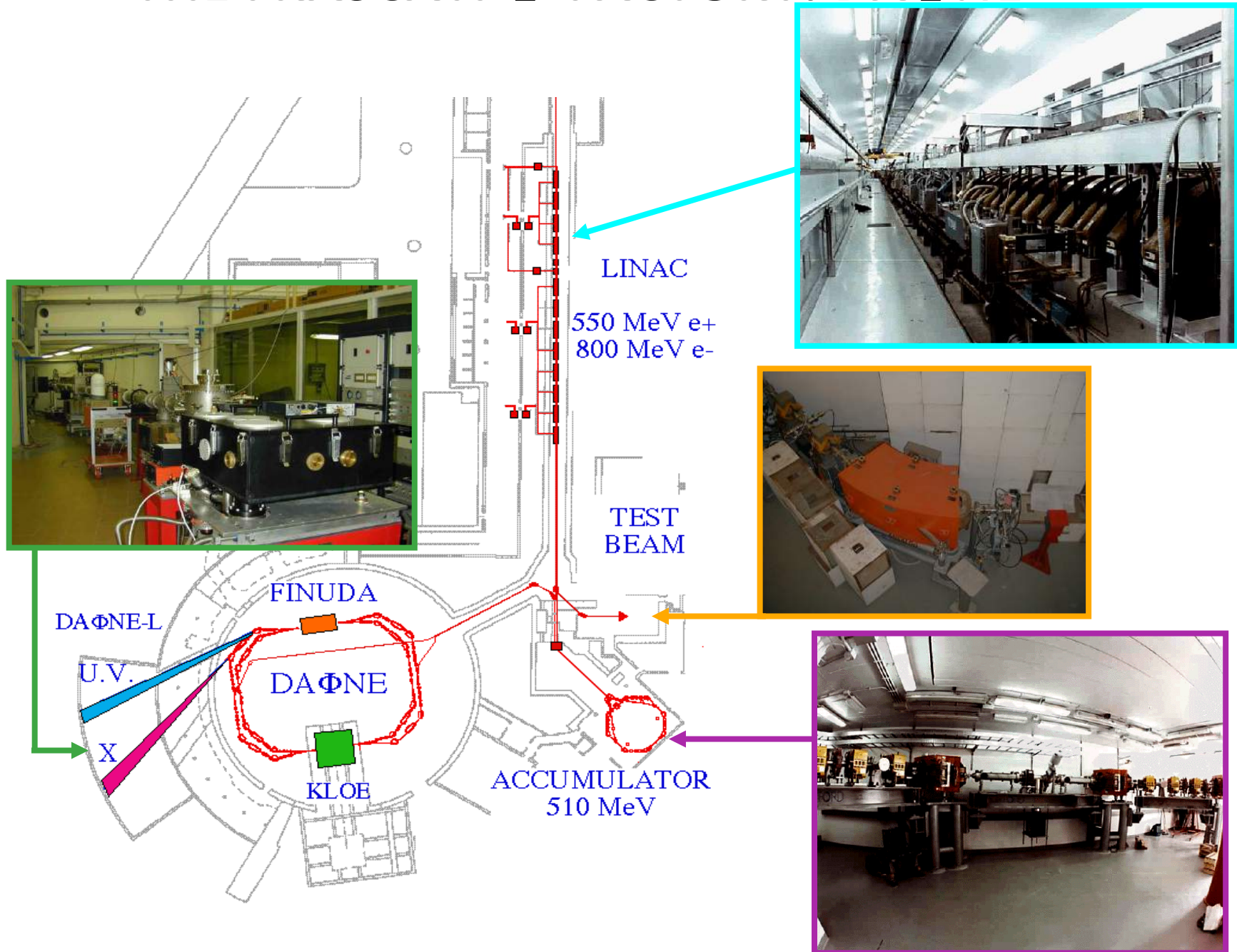
From the success of Ada it was decided to build a ring at higher energy (1.5 GeV per beam, 105 m): ADONE.



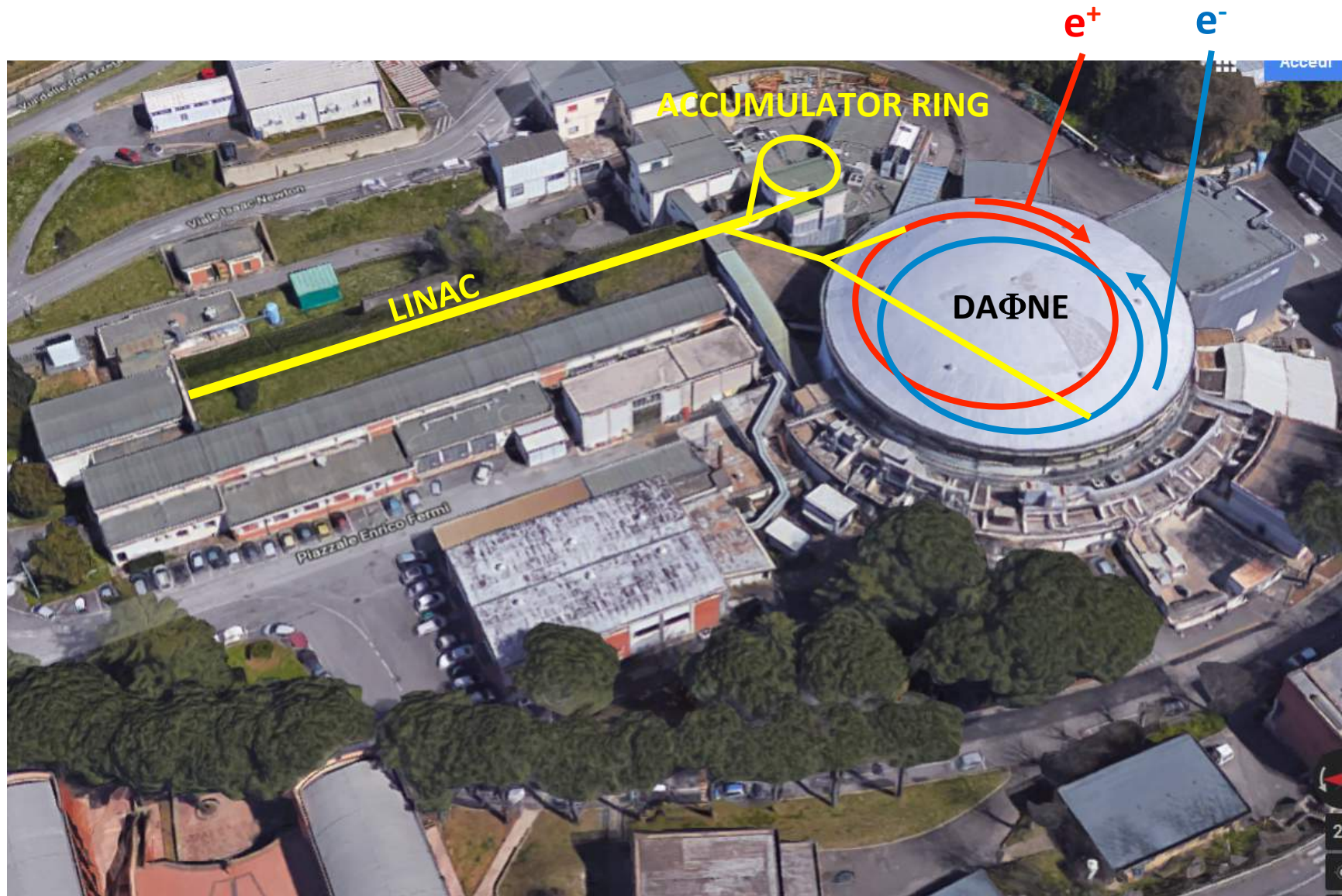
The construction of the new machine began in **1963** and the first electron was stored in 1967. A LINAC of 350 MeV was used as an injector. The maximum current circulating in ADONE was **100 mA in 3 bunches**. The luminosity target was reached:  $3 \times 10^{29} \text{ cm}^{-2} \text{ s}^{-1}$ . ADONE was **switched off the 26 April 1993**.



# THE FRASCATI $\Phi$ -FACTORY: DAΦNE



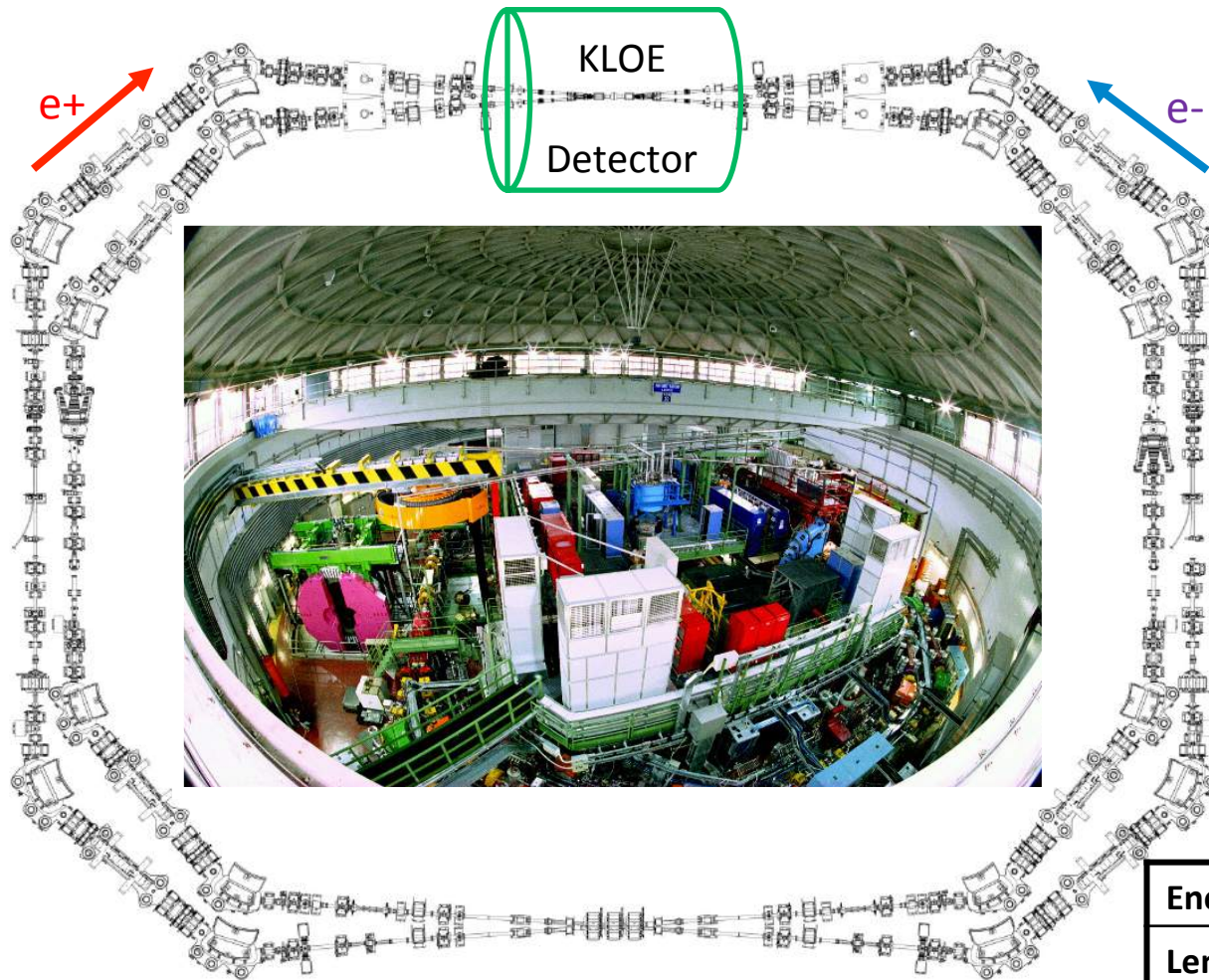
# $\Phi$ -FACTORY COMPLEX



Google image



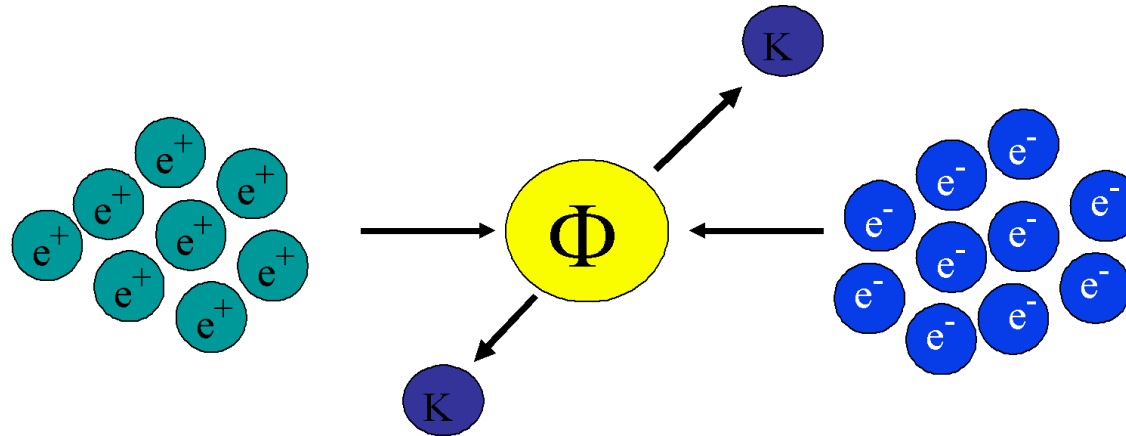
# FEW DAΦNE DAFNE PARAMETERS



<b>Energy</b>	510 MeV/beam
<b>Length</b>	96 m
<b><math>f_{RF}</math></b>	368 MHz
<b>Number of bunches</b>	120/ring
<b>Bunch length</b>	1.5-2.5 cm
<b><math>I_{MAX}</math></b>	2.5 A (ring $e^-$ )

# PARTICLE PHYSICS @DAΦNE

From the  $e^-/e^+$  collision at the energy of 1.02 GeV the  $\Phi$  particle is produced. This particle decays in kaons (K).



Kaons are used in the experiments: KLOE (KLOE 2), FINUDA, DEAR e SIDDHARTA (SIDDHARTA 2)

**KLOE**  
(K LOng Experiment)

KLOE studia il rapporto tra materia e antimateria tramite i decadimenti dei Kaoni

**FINUDA**  
(Fisica Nucleare a DAΦNE)

L'esperimento FINUDA studia la *forza forte* attraverso l'inserimento di un "corpo estraneo" all'interno del *nucleo*

**Ipernucleo**

$K^- \rightarrow \Lambda + \pi^-$

Ecco come appare un evento ipernucleare all'interno del rivelatore

di macchine acceleratrici - Acifrazza 11-15 Giugno 2007

**DEAR**  
(DAΦNE Exotic Atom Research)

**Idrogeno Kaonico**

Target cell  
cold finger  
Vacuum chamber  
Target-cell  
CCDs  
DEAR Beam pipe  
Kaon entrance window

$2p \rightarrow 1s (K_{\alpha})$   
**X ray of interest**

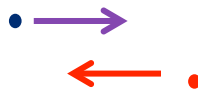
L'esperimento DEAR studia la *forza forte* attraverso lo studio degli *atomi kaonici* (in cui un  $K^-$  ha sostituito un elettrone atomico).



# CROSS SECTION AND COLLIDER LUMINOSITY

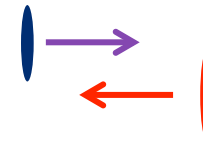
Two particles ( $e^+/e^-$  in DAΦNE, for example) colliding can produce **different types of events**, with different **probability**. The cross section  $\sigma$  of a given event is proportional to the probability that the event occurs and is measured in  $\text{cm}^2$ .

With respect to a given event, everything goes as if the **particles are small disks** with a finite "area"  $\sigma$  (measured in  $\text{cm}^2$ ). The interaction occurs if the two particles "touch" each other.



**Event with low probability.**

Low cross section



**Event with high probability.**

High cross section

The **cross sections are typically very small**, in fact the unit of measurement of the area  $\sigma$  is the **barn**. Dimensionally one barn is an area equal to  $10^{-28} \text{ m}^2$  or  $10^{-24} \text{ cm}^2$

In a collider, the frequency with which a given events occur can be expressed as the product  $\mathcal{L}\sigma$  where  $\mathcal{L}$  is called **collider luminosity**.



It is possible to demonstrate that **luminosity** of a collider can be calculated by the overlay integral of the two beams of particles in the 4-dimensini  $(x, y, z, t)$ . It is, in conclusion, a measure of how many beam-beam interactions we are producing

$$\mathcal{L} = f_c \int \int \int \int_{-\infty}^{+\infty} \rho^+(x, y, s + ct) \rho^-(x, y, s - ct) 2cdt ds dx dy$$

# LUMINOSITY

The luminosity is related to:

- the amount of colliding particles
- frequency of collision
- the beams dimensions at the interaction point

For gaussian beams

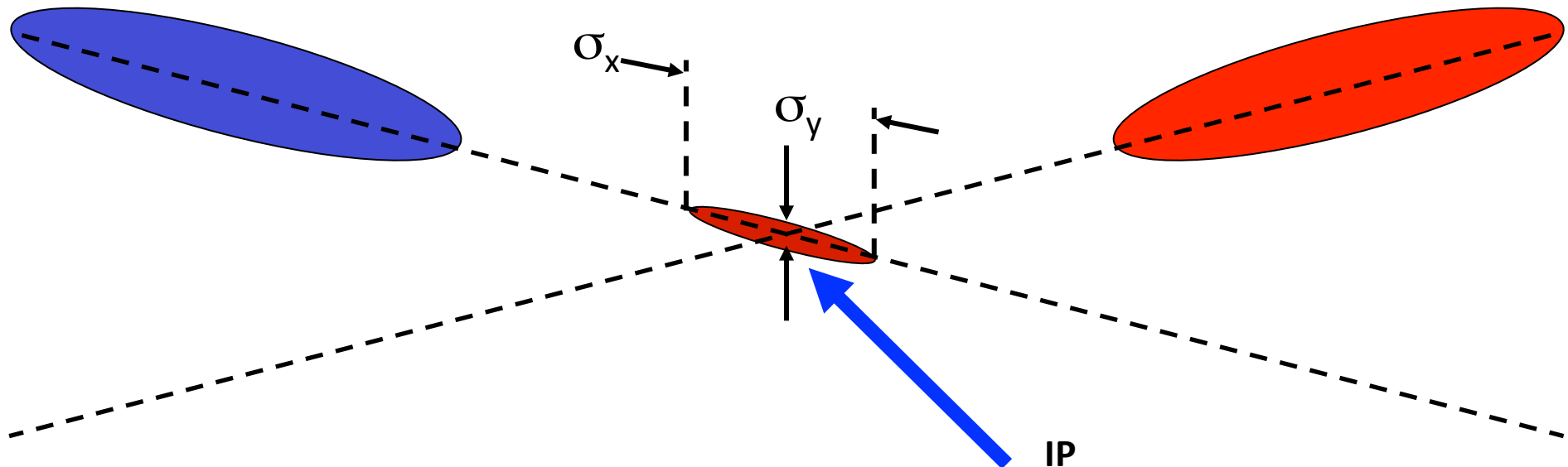


$$L = \frac{N^{e+} N^{e-}}{4\pi\sigma_x \sigma_y} \times f_{collision} \quad [cm^{-2} sec^{-1}]$$

Number of particles per beam ( $\sim 10^{10}$ )

Collision frequency (up to hundreds of MHz)

Transverse dimensions at the Interaction point (IP): up to few  $\mu$  (or even less)



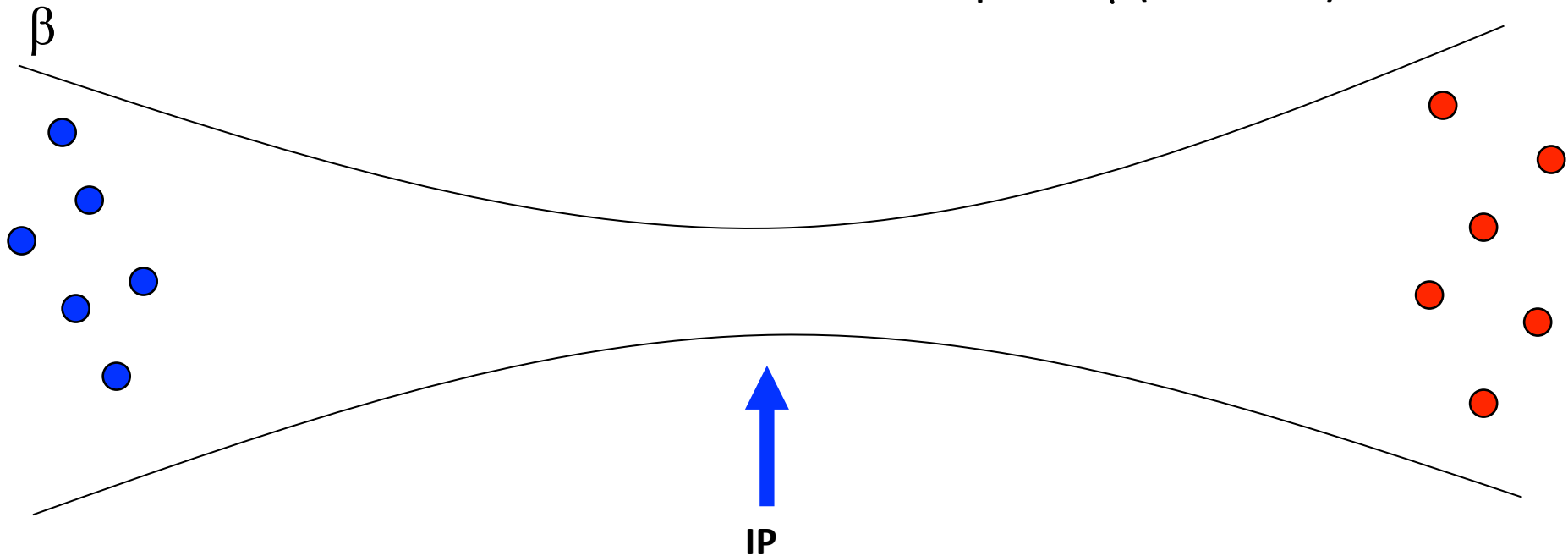
# LUMINOSITY

Number of particles  
per beam ( $\sim 10^{10}$ )

Collision frequency (up  
to hundreds of MHz)

$$L = \frac{N^{e^+} N^{e^-}}{4\pi\sigma_x \sigma_y} \times f_{\text{collision}} \quad [cm^{-2} sec^{-1}]$$

Transverse dimensions at the Interaction point (IP):  
up to few  $\mu$  (or even less)



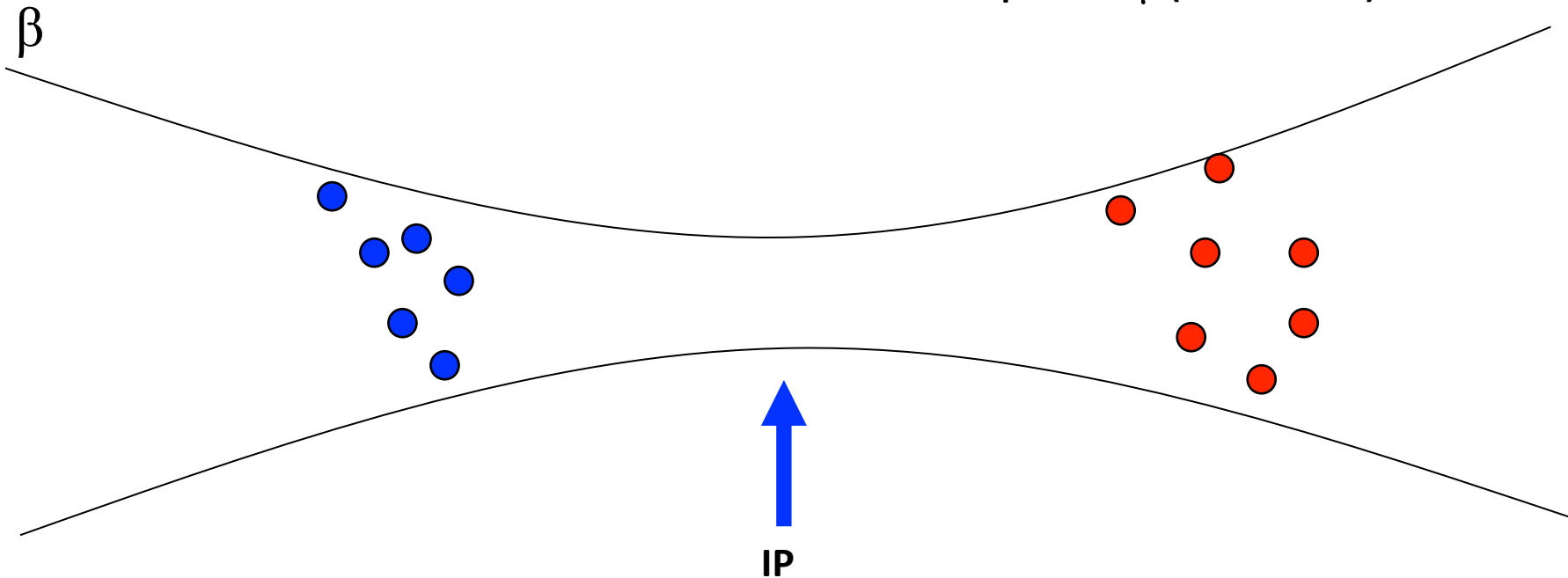
# LUMINOSITY

Number of particles  
per beam ( $\sim 10^{10}$ )

Collision frequency (up  
to hundreds of MHz)

$$L = \frac{N^{e^+} N^{e^-}}{4\pi\sigma_x \sigma_y} \times f_{\text{collision}} \quad [cm^{-2} sec^{-1}]$$

Transverse dimensions at the Interaction point (IP):  
up to few  $\mu$  (or even less)





# LUMINOSITY

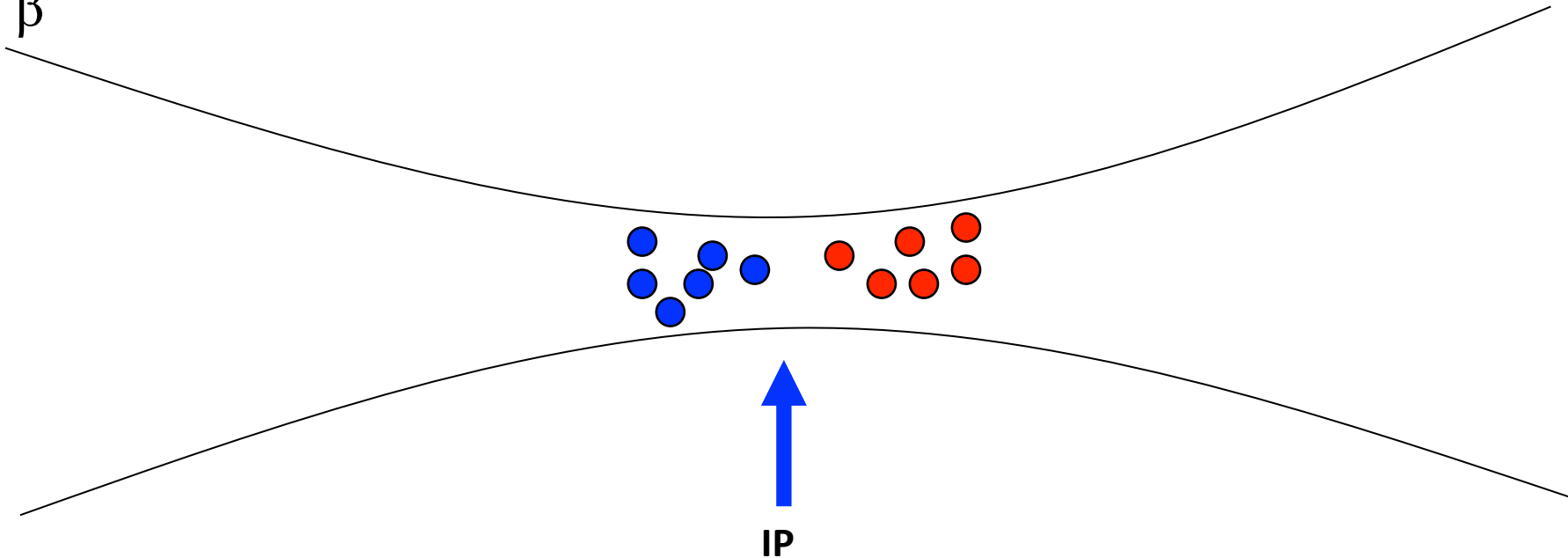
Number of particles  
per beam ( $\sim 10^{10}$ )

Collision frequency (up  
to hundreds of MHz)

$$L = \frac{N^{e^+} N^{e^-}}{4\pi\sigma_x \sigma_y} \times f_{\text{collision}} \quad [cm^{-2} sec^{-1}]$$

Transverse dimensions at the Interaction point (IP):  
up to few  $\mu$  (or even less)

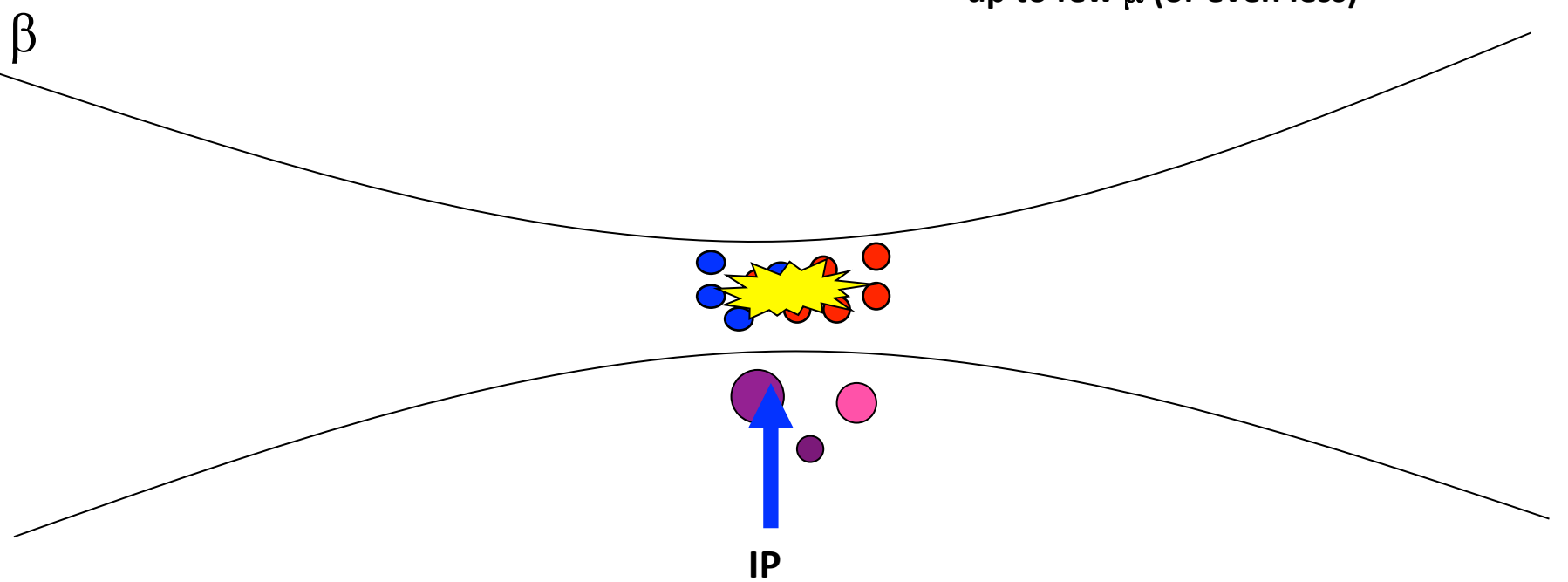
$\beta$



# LUMINOSITY

Number of particles  
per beam ( $\sim 10^{10}$ )

Collision frequency (up  
to hundreds of MHz)


$$L = \frac{N^{e+} N^{e-}}{4\pi\sigma_x \sigma_y} \times f_{collision} \quad [cm^{-2} sec^{-1}]$$

Transverse dimensions at the Interaction point (IP):  
up to few  $\mu$  (or even less)

$\beta$

IP

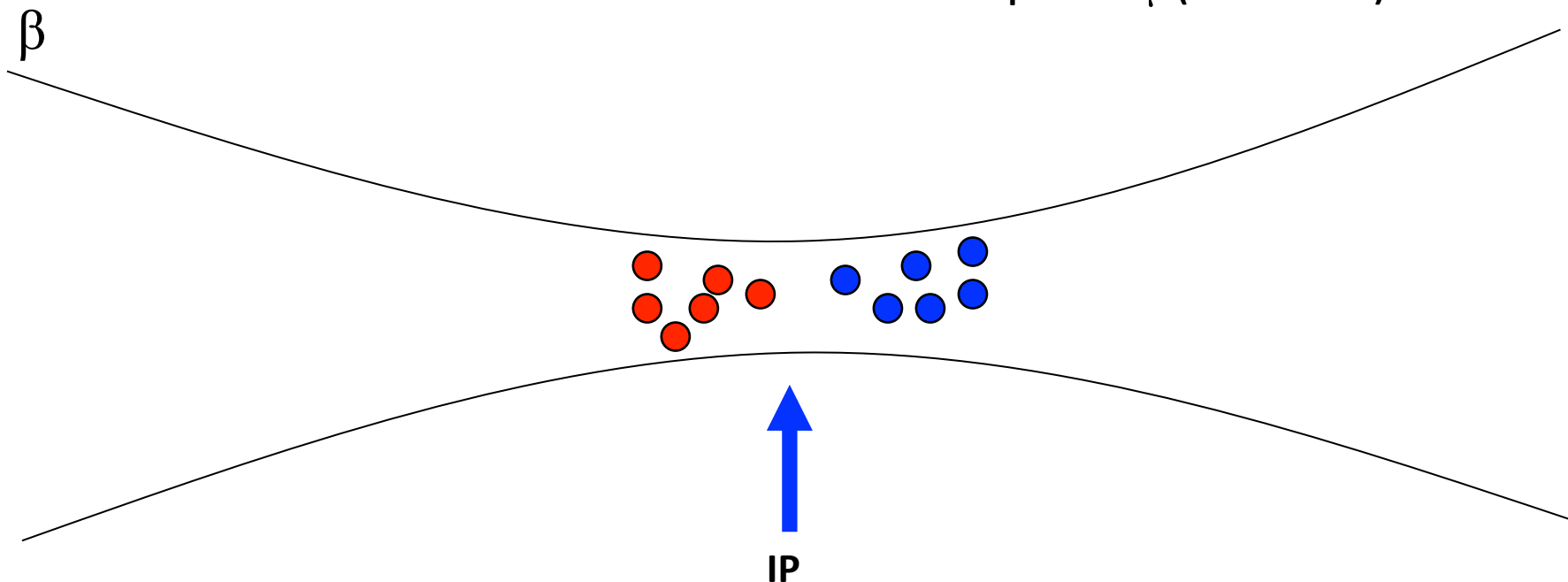
# LUMINOSITY

Number of particles  
per beam ( $\sim 10^{10}$ )

Collision frequency (up  
to hundreds of MHz)

$$L = \frac{N^{e^+} N^{e^-}}{4\pi\sigma_x \sigma_y} \times f_{\text{collision}} \quad [cm^{-2} sec^{-1}]$$

Transverse dimensions at the Interaction point (IP):  
up to few  $\mu$  (or even less)



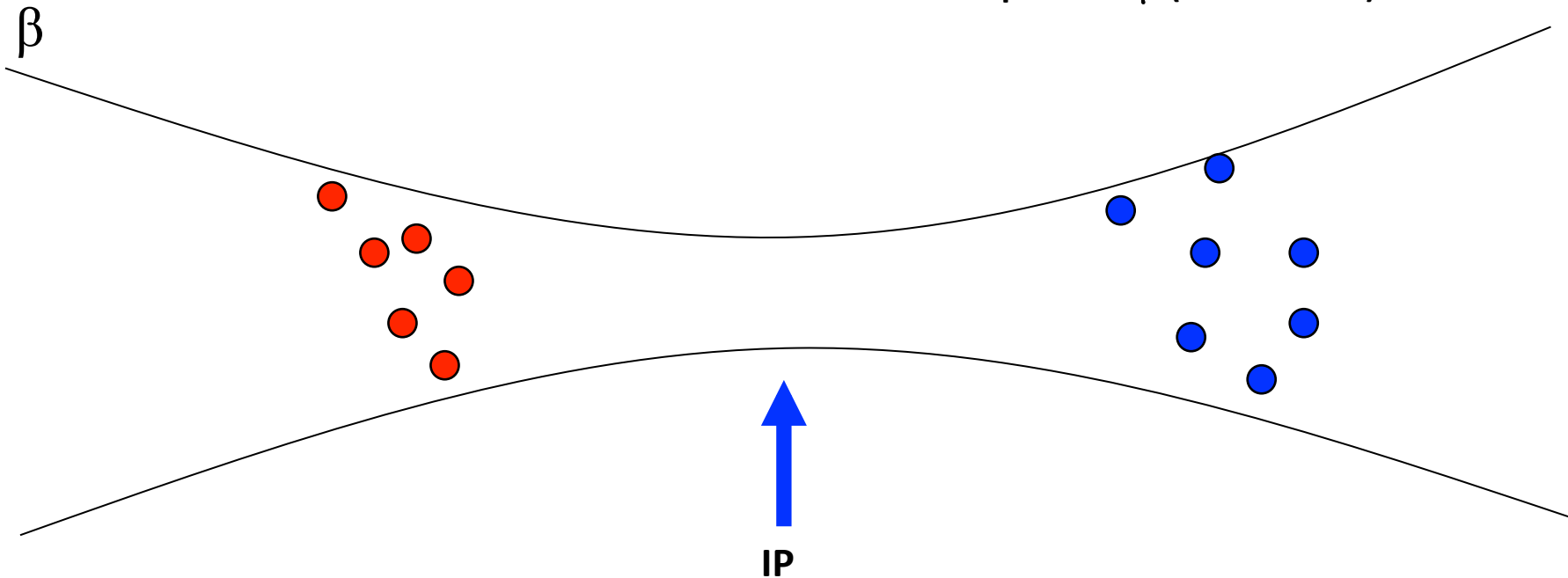
# LUMINOSITY

Number of particles  
per beam ( $\sim 10^{10}$ )

Collision frequency (up  
to hundreds of MHz)

$$L = \frac{N^{e^+} N^{e^-}}{4\pi\sigma_x \sigma_y} \times f_{\text{collision}} \quad [cm^{-2} sec^{-1}]$$

Transverse dimensions at the Interaction point (IP):  
up to few  $\mu$  (or even less)





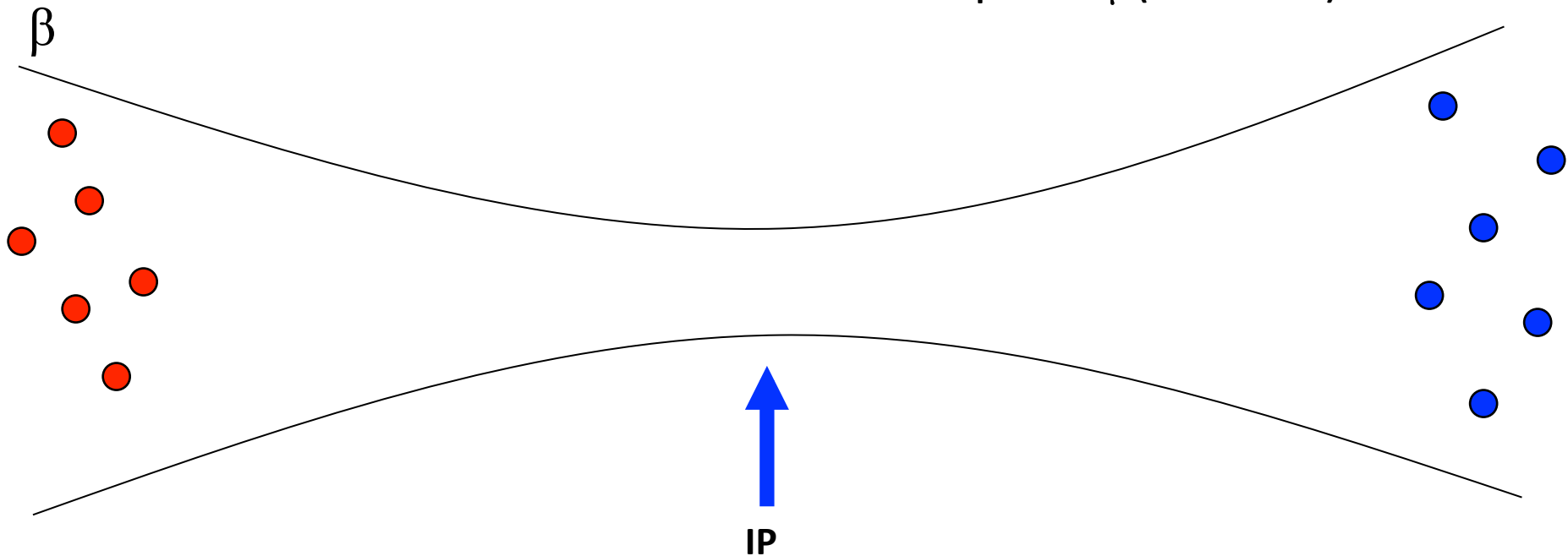
# LUMINOSITY

Number of particles  
per beam ( $\sim 10^{10}$ )

Collision frequency (up  
to hundreds of MHz)

$$L = \frac{N^{e^+} N^{e^-}}{4\pi\sigma_x \sigma_y} \times f_{\text{collision}} \quad [cm^{-2} sec^{-1}]$$

Transverse dimensions at the Interaction point (IP):  
up to few  $\mu$  (or even less)



# LUMINOSITY: EXAMPLE

Particle  $\Phi$  generation @ DAΦNE

$$L = \frac{\overset{\sim 2 \cdot 10^{10}}{N^{e^+}} \overset{\sim 2 \cdot 10^{10}}{N^{e^-}}}{4\pi \underset{1 \text{ mm}}{\sigma_x} \underset{10 \mu\text{m}}{\sigma_y}} \times \underset{368.000.000 \text{ Hz (120 bunches)}}{f_{\text{collision}}} \cong 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$$

Cross section  $\Phi$   
 $\sigma_\Phi \sim 3 \cdot 10^{-30} \text{ cm}^2$



frequency of event of  $\Phi$  particle production  
 $L\sigma = 300 \text{ events/s}$

# LEP (LARGE ELECTRON POSITRON) CERN 1988-2001

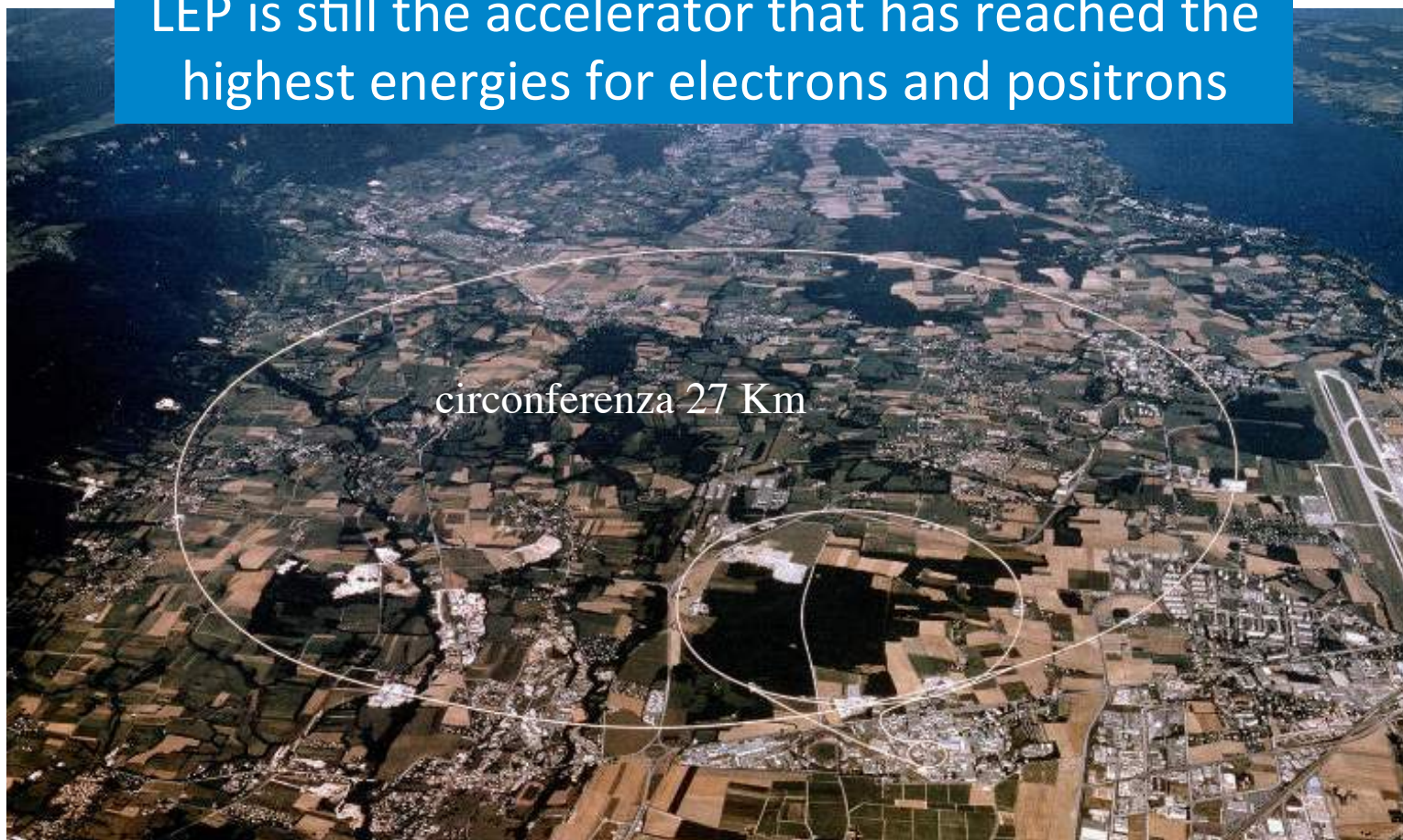
## LEP1

1300x10<sup>6</sup> CHF    accelerator and infrastructure cost  
1989              First collisions  $E_{\text{CM}} = 91 \text{ GeV}$  (Z0 energy)

## LEP2

1995              Superconducting cavities installation  $E_{\text{CM}} = 209 \text{ GeV}$

LEP is still the accelerator that has reached the highest energies for electrons and positrons



# LHC (LARGE HADRON COLLIDER) CERN

## LHC main parameters

Colliding particles proton-proton but also ions (Pb-Pb)

Max Energy per beam 7 TeV

Number of bunches 2808

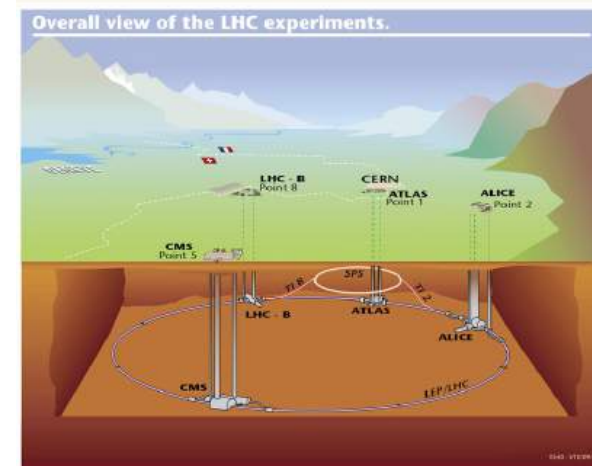
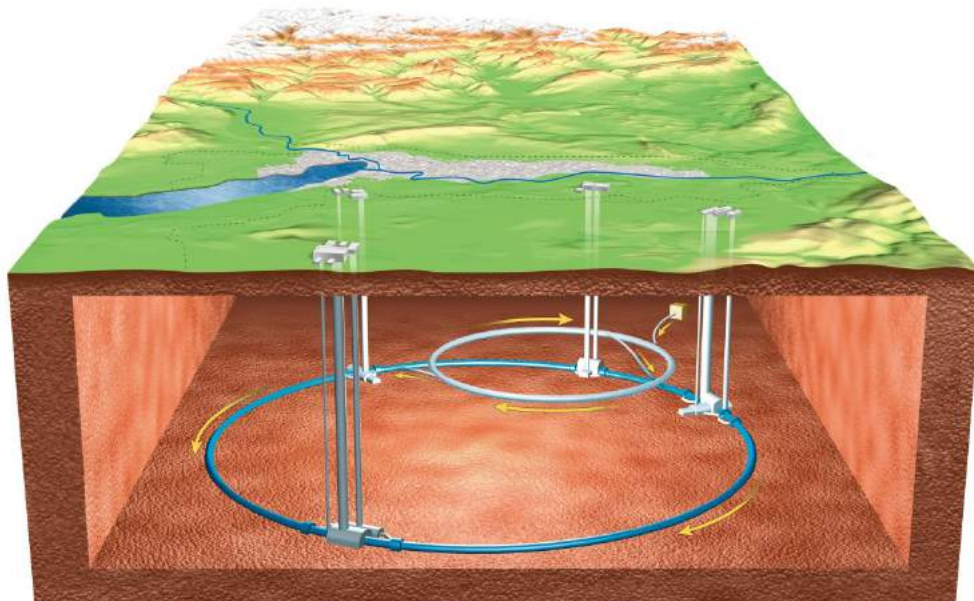
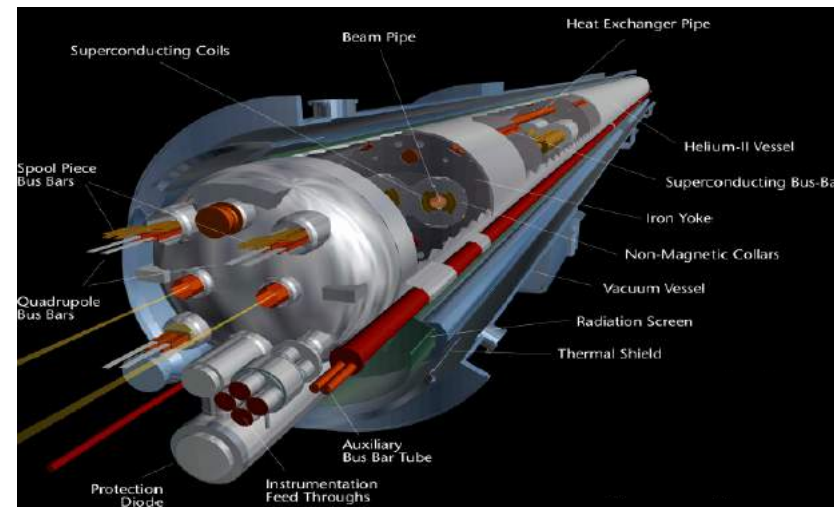
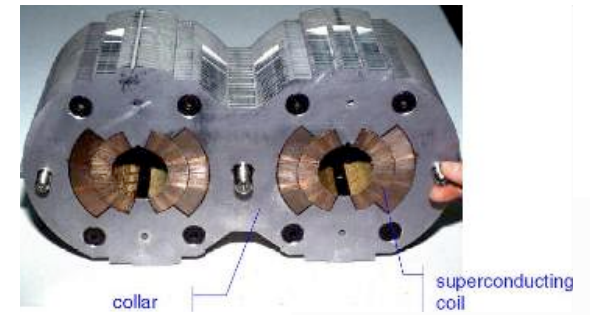
Crossing angle 300  $\mu$ rad

Emittance  $5 \times 10^{-10}$  m

IP transv. Dim. ( $\sigma_x = \sigma_y$ ) 16  $\mu$ m

Machine length 27.8 Km

$B_{MAX}$  dipoles  $\sim 8$  T with  $I = 11700$  A @  $T = 1.9$  K





# LHC TUNNEL

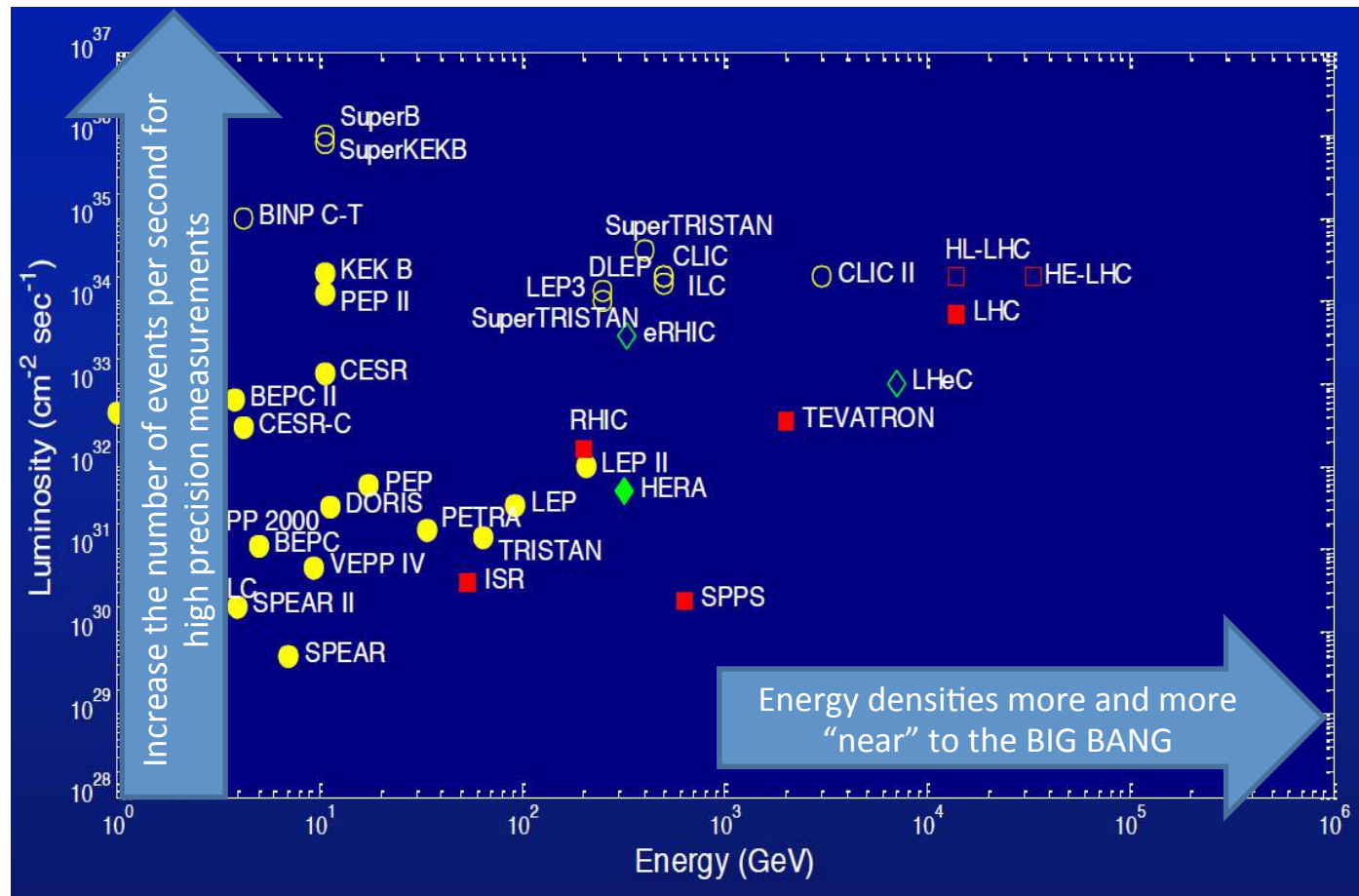


# ENERGY AND LUMINOSITY: COLLIDER DEVELOPMENT

The development of colliders followed two different directions:

-**higher and higher energies** (discovering machines, LHC,...)

-**higher and higher luminosities** (to increase the number of events and to perform precise measurements)







# LIMITS OF HIGH ENERGY CIRCULAR MACHINES

Hadron colliders (p,...)

$$\rho \cong \frac{p[GeV/c]}{0.3B}$$

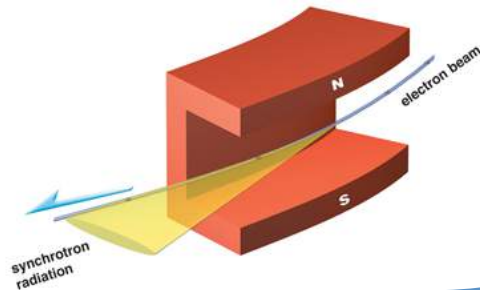


The maximum magnetic fields reachable with superconducting magnets could be 18-20 T (8 T LHC)

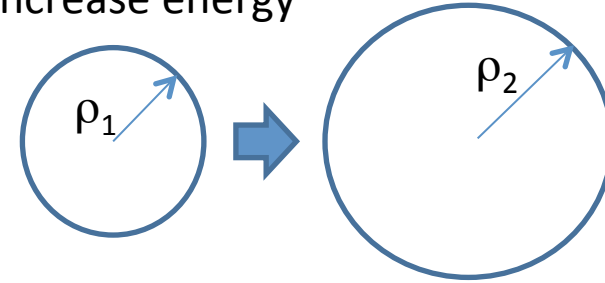
Lepton colliders

The limit even more than the maximum radius of curvature is given by the **power lost by synchrotron light emission**

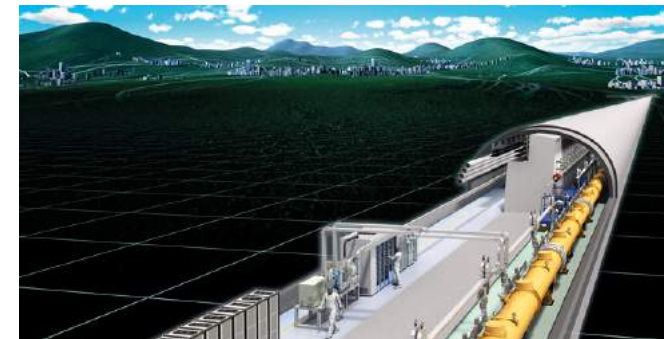
$$U_{per\ turn} \propto \frac{\gamma^4}{\rho}$$



Larger and larger machines are required to increase energy



**linear accelerator** with high accelerating field





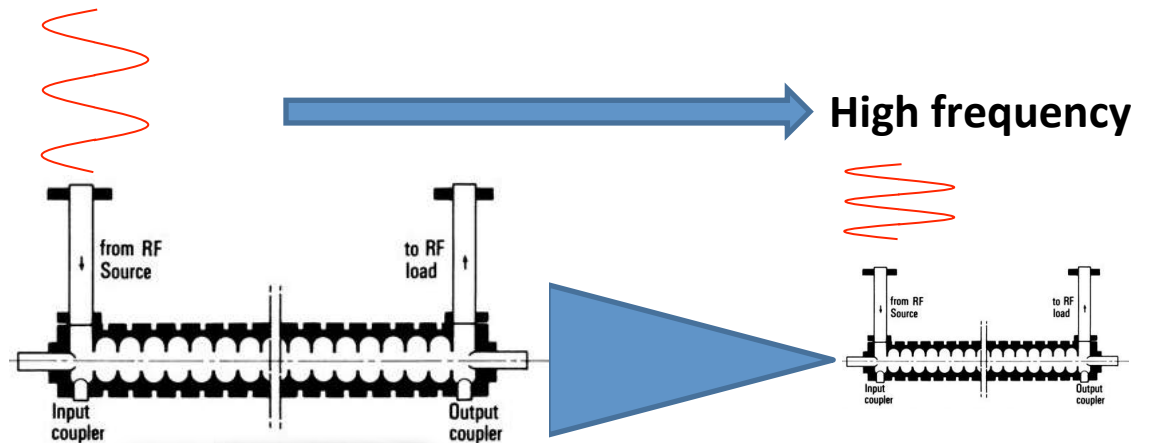
# HIGH ACCELERATING FIELD $\Rightarrow$ HIGH FREQUENCY

The basic idea is to **concentrate electromagnetic energy** in smaller volumes to increase its density and, therefore, the value of the accelerating field.

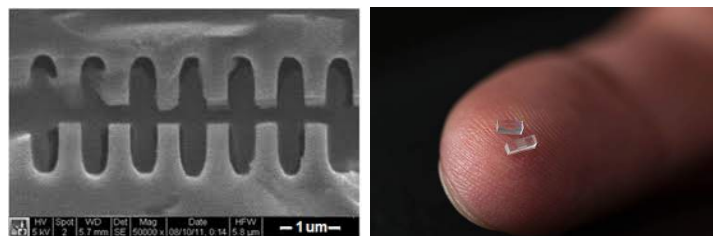


... Compatibly with:

- Available Electromagnetic **sources**
- **Dissipation** on the structures
- Discharges limits (**breakdown**)



**METALLIC STRUCTURES**  
Fed by high power **klystrons**

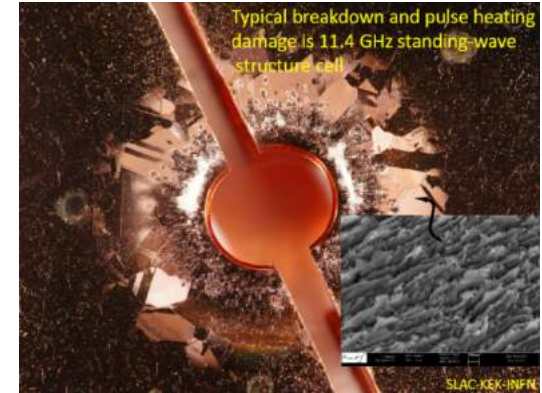
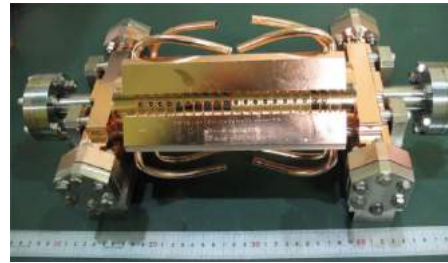


**DIELECTRIC STRUCTURES**  
Fed by **lasers**



# METALLIC STRUCTURES

Metallic structures working at **12 GHz** can reach up to **100-150 MV/m** accelerating field gradient with a small number of discharges. At higher frequencies commercial power sources are not available.

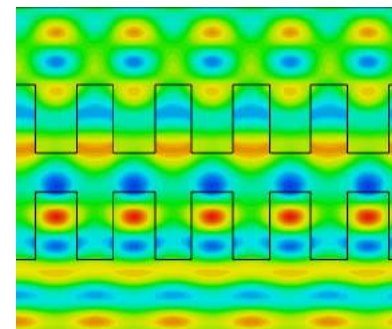
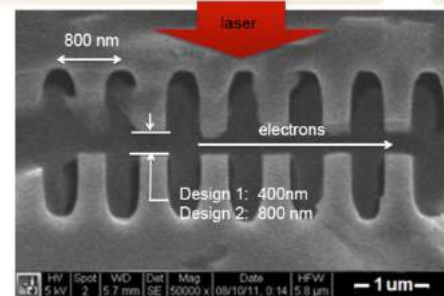
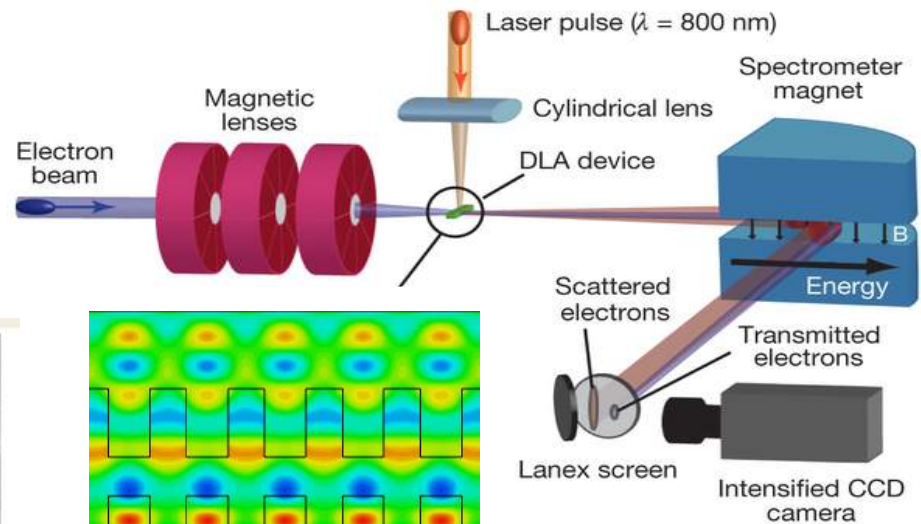


# DIELECTRIC STRUCTURES POWERED BY LASERS

**Lasers** are intense pulsed sources of electromagnetic field and at the working frequencies of lasers ( $10^{13}$ - $10^{15}$ Hz) metallic structures cannot be used (dissipation,...)



Dielectric structures can be used



Gradient up to **1 GV/m** have been measured

# PROBLEMS RELATED TO HIGH FREQUENCY STRUCTURES

High gradient (**100 MV/m**) **X-band LINAC** can be designed and fabricated but, in order to reach energies up to **1 TeV** a few km LINAC is necessary.

On the other hand **miniaturized structures** (dielectric) allow in principle reaching even larger gradients but there are several problems still not solved like:

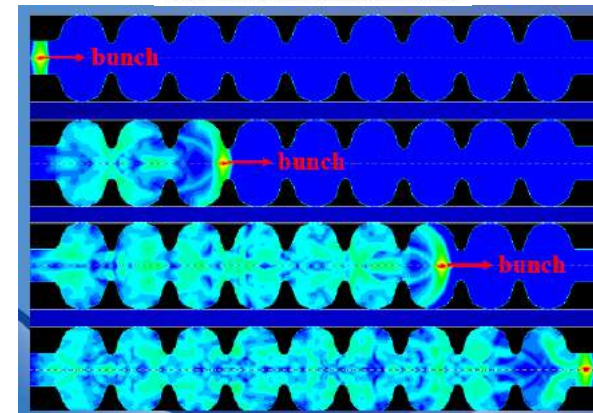
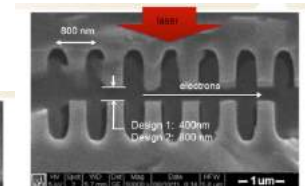
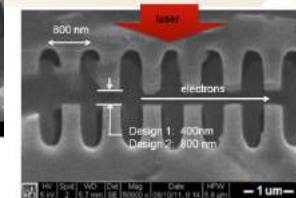
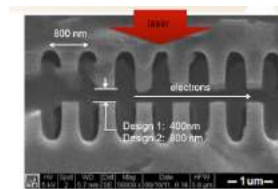
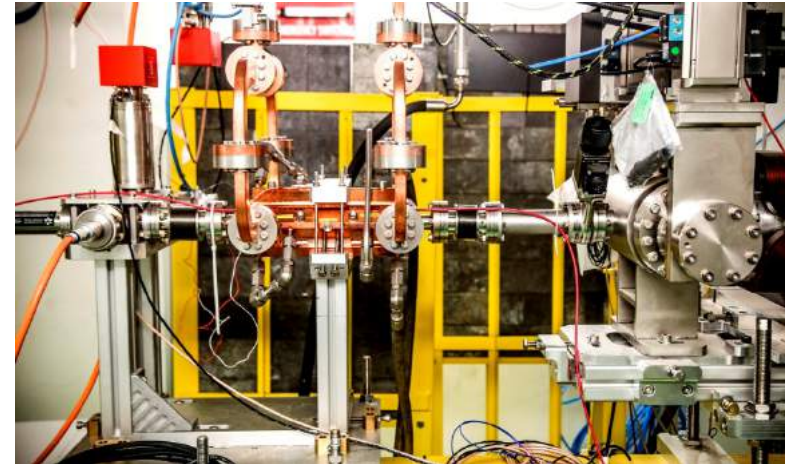
-the **amount of charge and beams dimensions** that is possible to accelerate

-**Synchronization** and total **available length for acceleration**: remember that the gradient is important but also the total length of the structure (a 1mm structure at 1GV/m gives 1MeV energy gain!)

-**Quality of the accelerating field** (with respect to RF structures)

-**Alignment and stabilization problems**

-**wakefield instabilities**





# PLASMA ACCELERATORS (see M. Ferrario lecture)

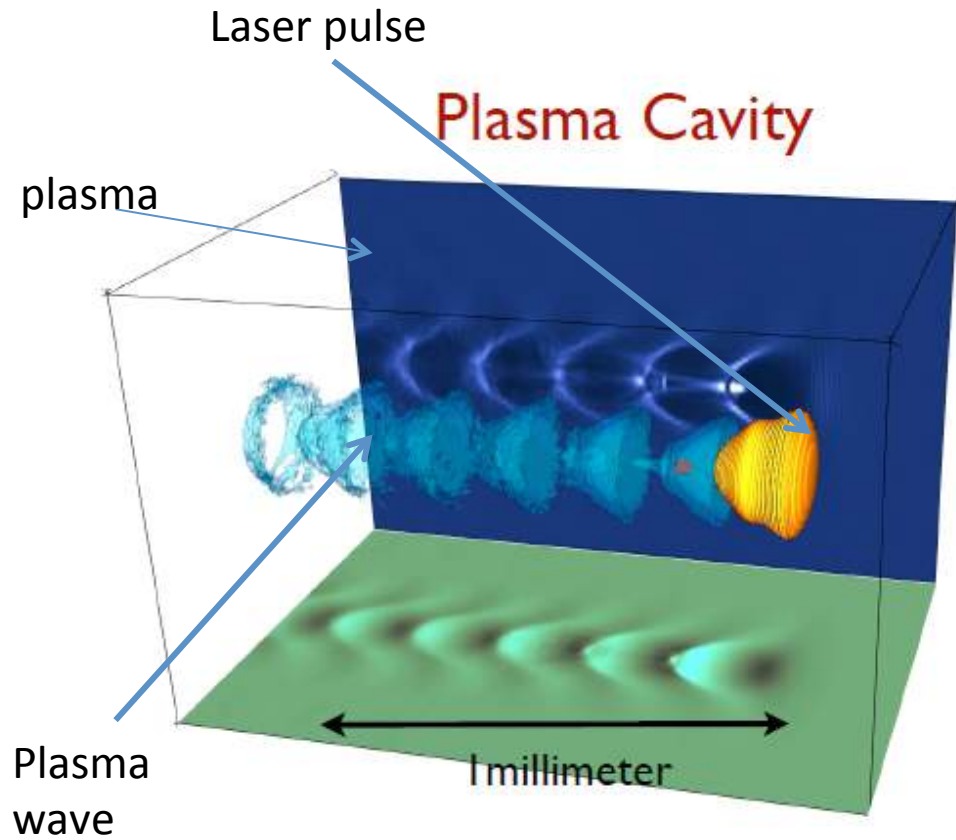
RF Cavity



1 meter

Electric field <math>< 100 \text{ MV/m}</math>

Plasma Cavity

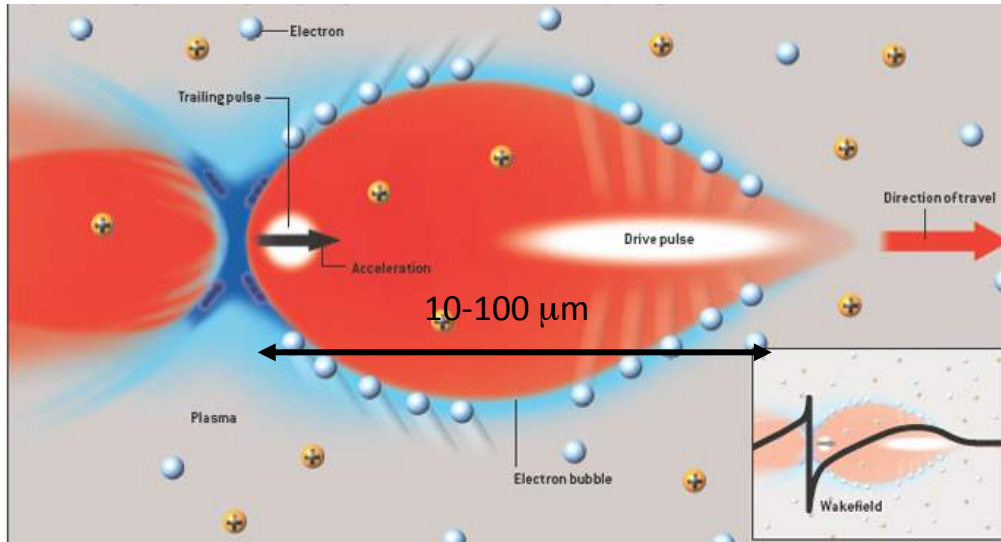


Field up to  $100 \text{ GV/m}$

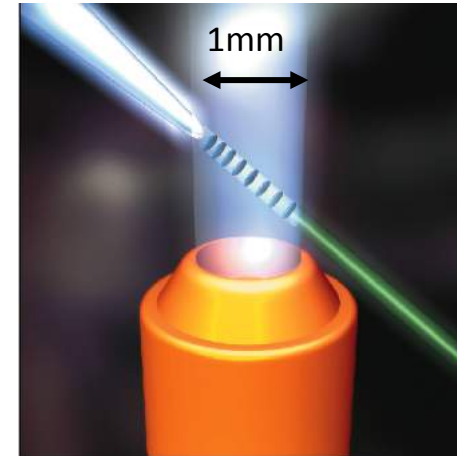
km  m



# ACCELERAZIONE LASER-PLASMA (LWFA)



laser Gas jet



VIDEO

Elettroni accelerati

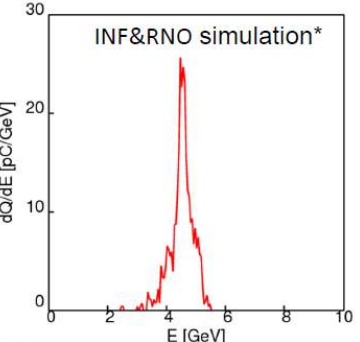
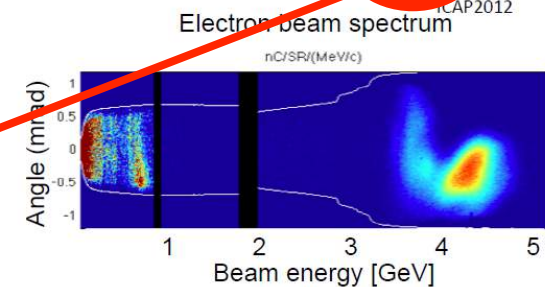
$$E_0 = \frac{m_e c \omega_p}{e} \approx 100 \left[ \frac{\text{GeV}}{m} \right] \cdot \sqrt{n_0 [10^{18} \text{ cm}^{-3}]}$$

4.25 GeV beams have been obtained from 9 cm plasma channel powered by 310 TW laser pulses (15 J)

\*C. Benedetti et al., proceedings of AAC2010, proceedings of CAP2012



Leader mondiale: BELLA Center, Accelerator Technology and Applied Physics Division  
Lawrence Berkeley National Laboratory



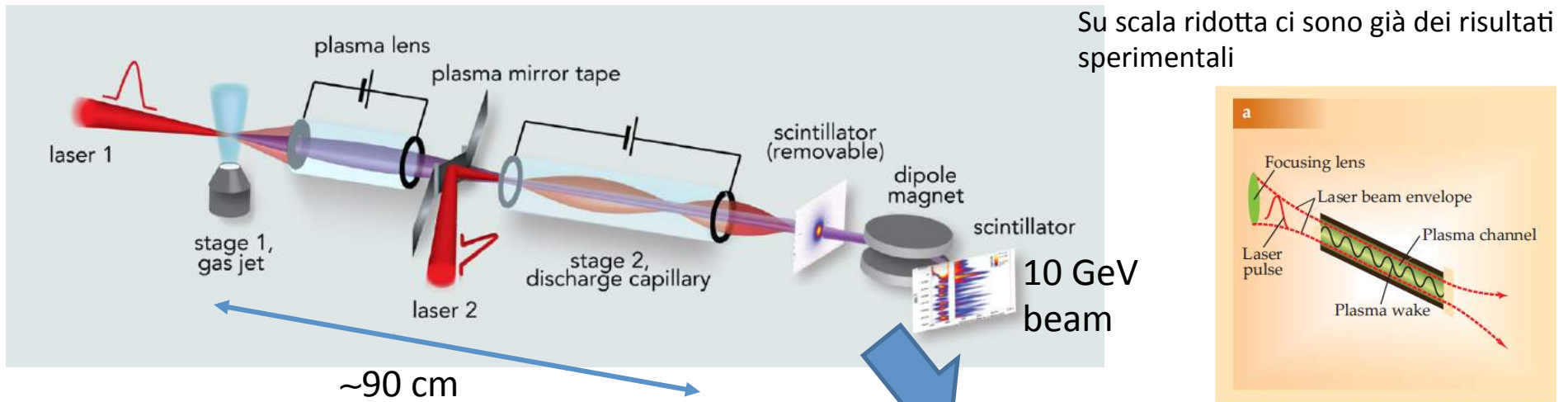
$$E_{\text{acc}} \cong 50 \text{ GV/m}$$

- **Laser** (E=15 J):
  - Measured) longitudinal profile ( $T_0 = 40 \text{ fs}$ )
  - Measured far field mode ( $w_0 = 53 \mu\text{m}$ )
- **Plasma**: parabolic plasma channel (length 9 cm,  $n_0 \sim 6\text{-}7 \times 10^{17} \text{ cm}^{-3}$ )

W.P. Leemans et al., PRL 2014

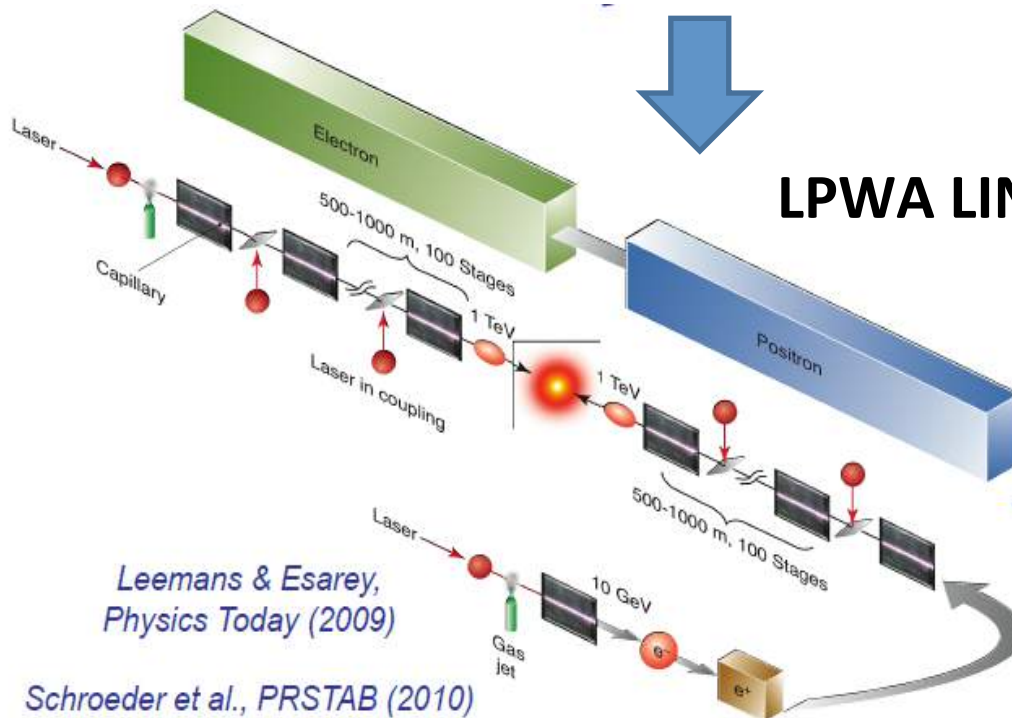
	Exp.	Sim.
Energy	4.25 GeV	4.5 GeV
$\Delta E/E$	5%	3.2%
Charge	$\sim 20 \text{ pC}$	23 pC
Divergence	0.3 mrad	0.6 mrad

# ACCELERAZIONE LASER-PLASMA: MULTI-GeV LINAC



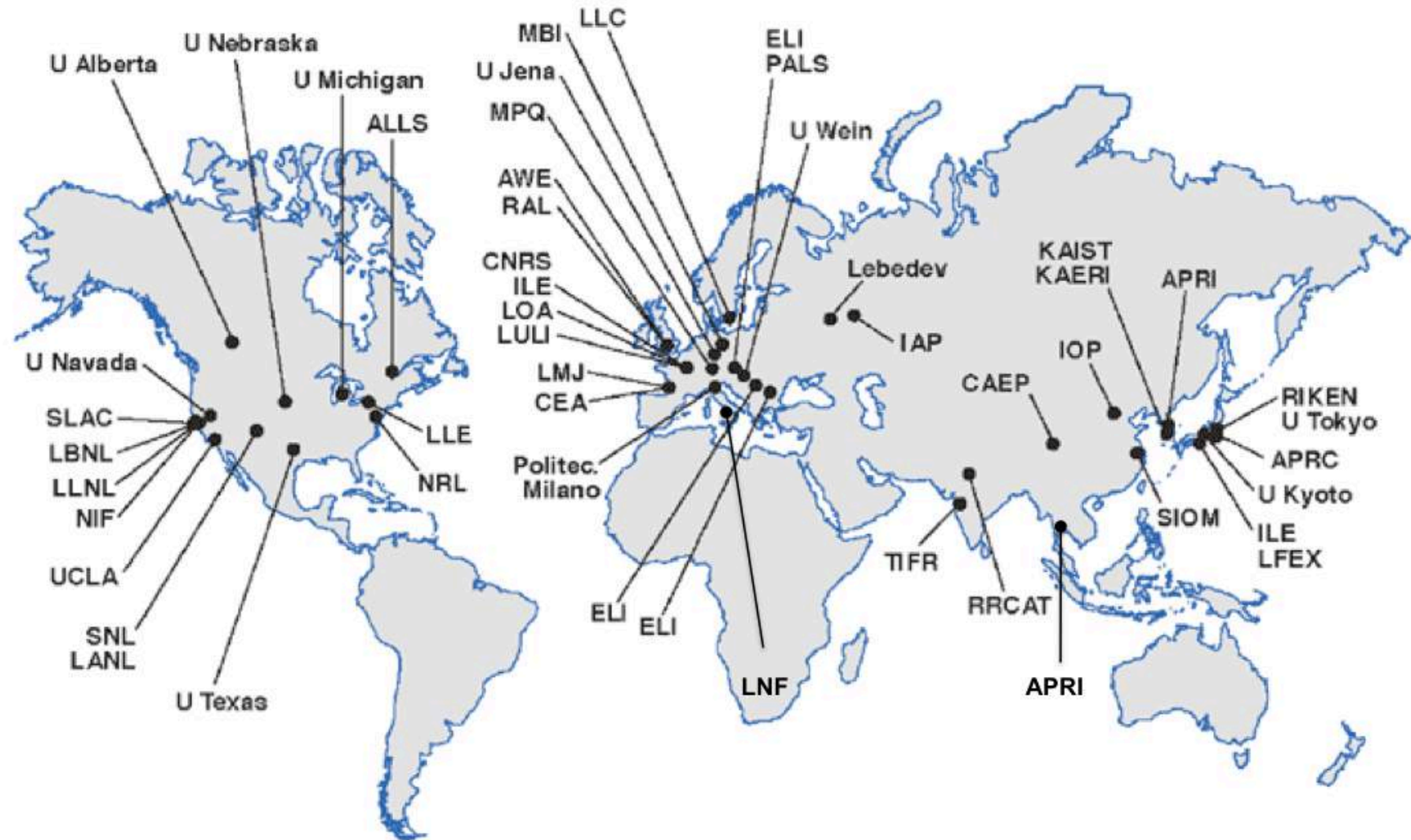
**SORGENTI DI RADIAZIONE COMPATTE BASATE SU LPWA**

**LPWA LINEAR COLLIDER?**



- LPA-linear collider:
- 50 stages (1 TeV collider)
- 10 GeV/stage
- requires ~10 J laser (at tens of kHz, hundreds of kW)
- $n=10^{17} \text{ cm}^{-3}$  (set by laser depletion)

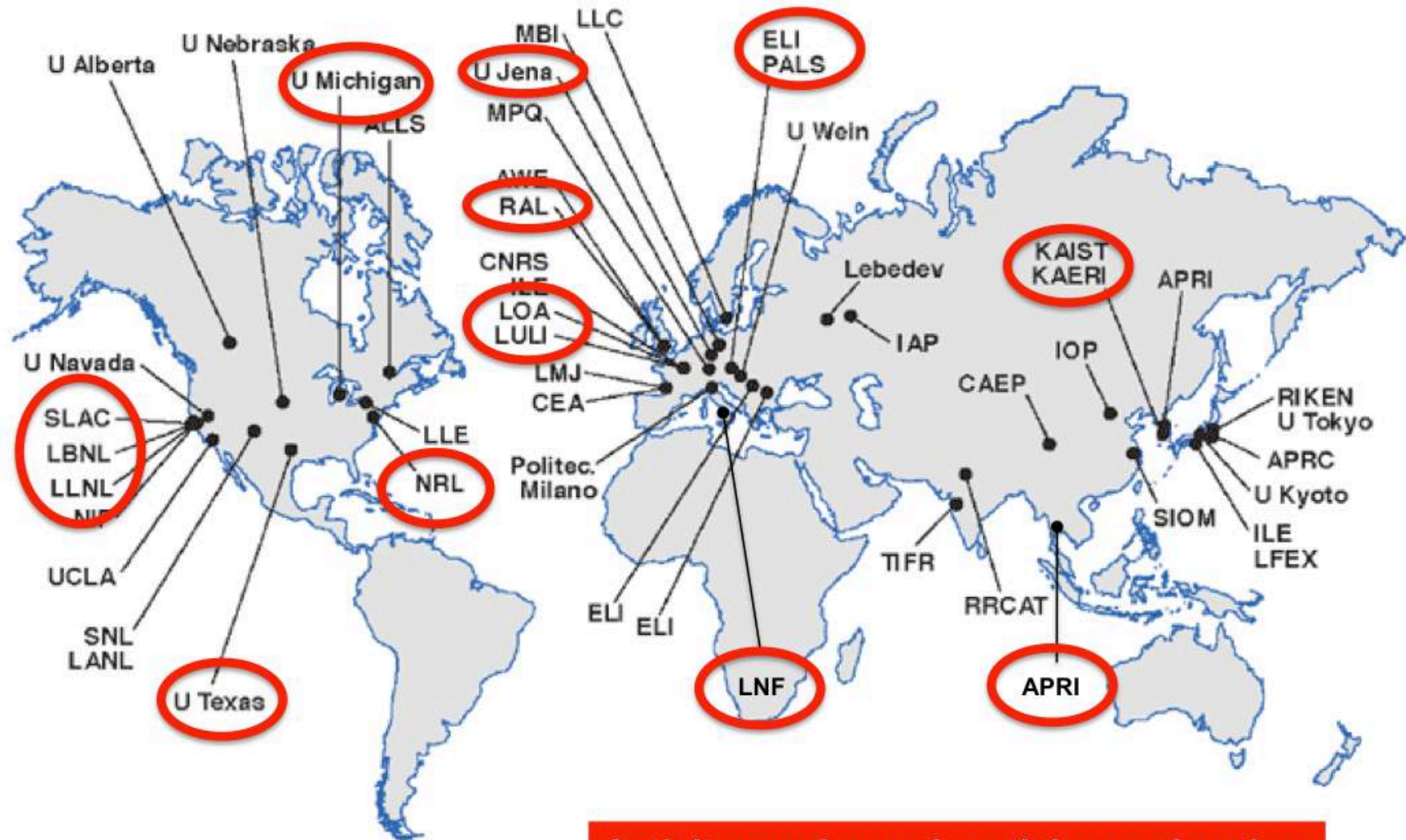
# ULTRA-HIGH INTENSITY LASER FACILITIES



<http://www.icuil.org/>



# ULTRA-HIGH INTENSITY LASER FACILITIES



<http://www.icuil.org/>

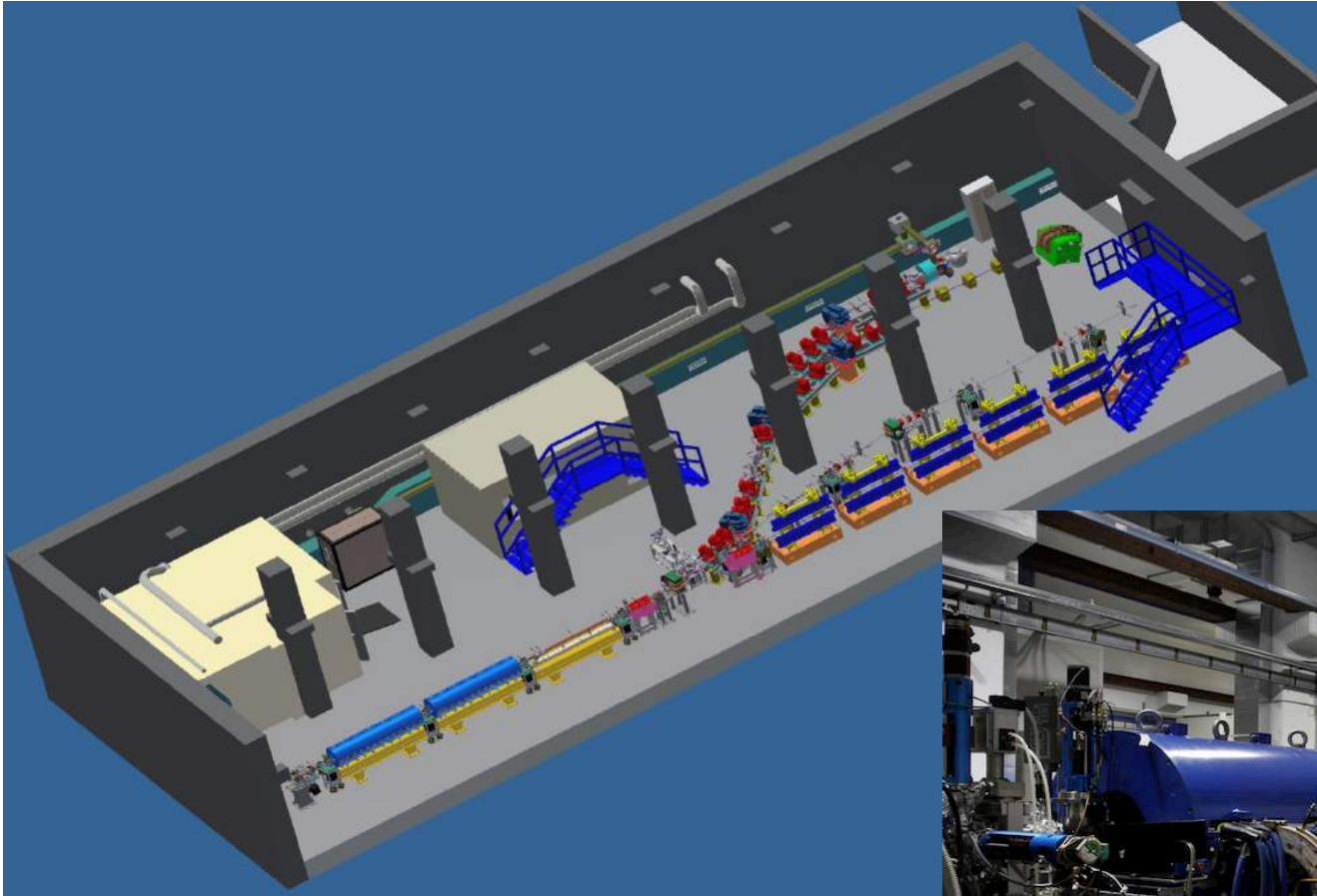
Activity on charged particle acceleration



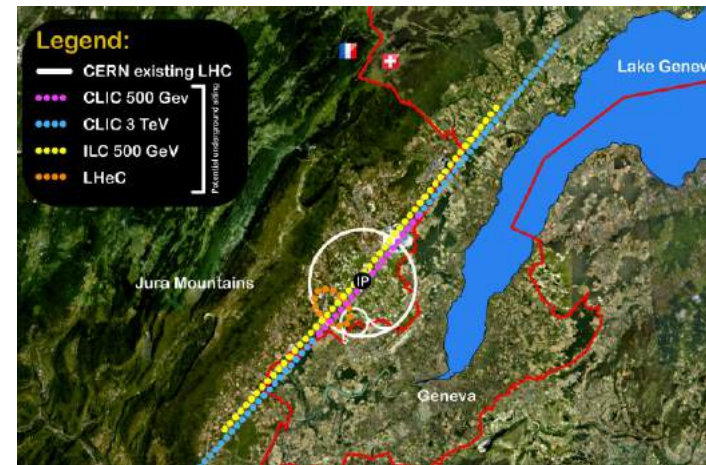
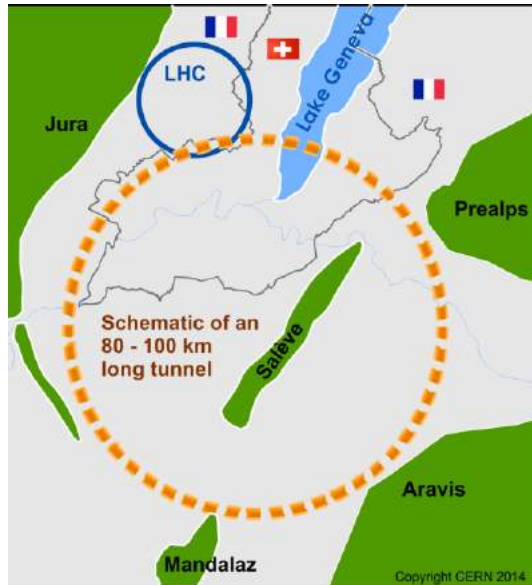
# SPARC\_LAB @LNF

*(Sources for Plasma Accelerators and Radiation Compton with Lasers And Beams)*

Anche ai LNF abbiamo un acceleratore dedicato ad esperimenti di: FEL, accelerazione al plasma, Generazione di radiazione THz e radiazione Compton.

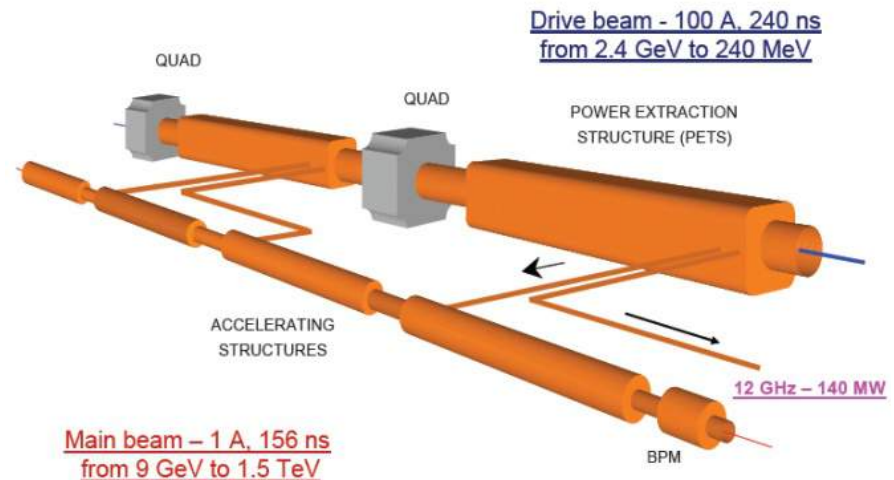


# STUDIES AT CERN ON FUTURE COLLIDERS

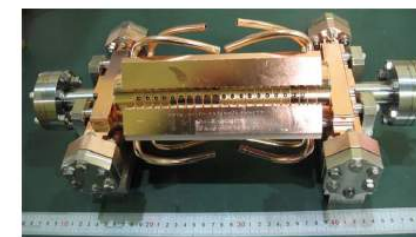


## FCC hadron collider parameters

Parameter	FCC-hh	SPPC	LHC	HL LHC
collision energy cms [TeV]	100	71.2	14	
dipole field [T]	16	20	8.3	
# IP	2 main & 2	2	2 main & 2	
bunch intensity [ $10^{11}$ ]	1	1 (0.2)	1.1	2.2
bunch spacing [ns]	25	25 (5)	25	25
luminosity/lp [ $10^{34} \text{ cm}^{-2}\text{s}^{-1}$ ]	5	~25	1	5
events/bunch crossing	170	~850 (170)	400	27
stored energy/beam [GJ]	8.4	6.6	0.36	0.7
synchrotron radiation [W/m/aperture]	30	58	0.2	0.35



12 GHz (X-band): 100 MV/m

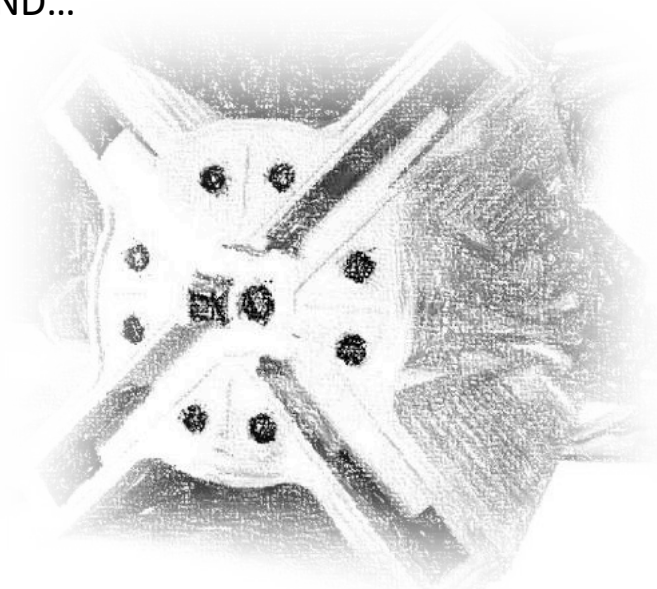


# CONCLUSIONS

⇒ **PARTICLE ACCELERATORS REMAIN ONE OF THE MOST POWERFUL INSTRUMENTS** IN FUNDAMENTAL PHYSICS RESEARCH, PHYSICS OF MATTER AND PARTICLES WITH MANY APPLICATIONS IN MEDICAL AND INDUSTRIAL FIELDS.

⇒ **A JUMP IN TECHNOLOGY IS NECESSARY** AND NEW IDEAS AND IMPORTANT RESULTS OPENED NEW AND PROMISING WAYS

⇒ **THE R&D IN THE PHYSICS AND TECHNOLOGY OF PARTICLE ACCELERATORS REMAINS ONE OF THE MOST EXCITING FIELDS OF RESEARCH IN APPLIED PHYSICS** IN WHICH **FANTASY AND CREATIVITY** ARE THE FUNDAMENTAL INGREDIENTS... AND...



**THANK YOU FOR YOUR ATTENTION!**

# ACKNOWLEDGEMENTS

M. E. Biagini, M. Boscolo, A. Cianchi, M. Ferrario, A. Gallo, S. Guiducci, C. Milardi, A. Mostacci, A. Balerna

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Animazione LINAC SLAC

<http://lcls.slac.stanford.edu/AnimationViewLCLS.aspx>

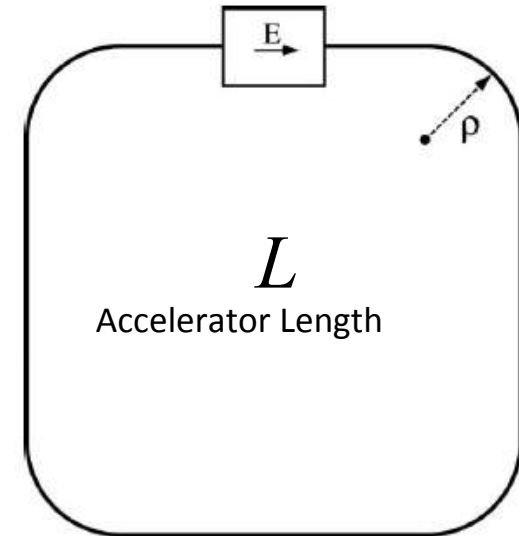


### 3) LONGITUDINAL DYNAMICS IN SYNCROTRONS

**Synchrotrons** are **circular accelerators**. Particles are bended by **dipolar magnets**, while acceleration is provided by **standing wave RF cavities**.

Differently from LINACs, in synchrotrons the particles run over closed orbits so that they are accelerated by the RF cavities in a very large number of turns.

Since the **beam energy** and **momentum** increase turn by turn, the bending **magnetic field** has to be **increased proportionally** to the **momentum**, in order to keep invariant the bending radii and the orbit. The revolution frequency also increases with energy, which requires variation of the RF frequency to maintain the synchronization between beam and accelerating field.



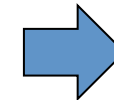
#### Synchronous particle and phase

The **synchronous particle** is the particle that undergoes the **ideal acceleration process**, gaining a **constant** amount of **energy  $\Delta W$  per turn** remaining all the time on the design orbit. While running the  $n^{\text{th}}$  turn, the synchronous particle has to have the proper energy to perfectly satisfy the following condition:

$$P_s|_n = qB|_n \rho$$

dipole magnetic field at turn # n      dipole bending radius, invariant during the whole acceleration process

The particle velocity increases turn by turn during acceleration (and tends asymptotically to the speed of light at high energies), and so it does the revolution frequency ( $F_{rev}$ ). As a consequence **the frequency of the RF system ( $f_{RF}$ )** must change to be synchronous with the beam motion.



Synchronization between accelerating field and synchronous particle requires:

$$f_{RF}|_n = h F_{rev-s}|_n$$

**harmonic number**

Corresponds to the number of time-separated synchronous particles that can be accelerated in the synchrotron

$$\Delta W_s|_n = \Delta W_s = q\hat{V} \cos\left(\underbrace{\int_0^{t_n} 2\pi f_{RF} dt + \varphi_0}_{\phi_s}\right)$$

Energy gain of the synchronous particle due to the  $n^{\text{th}}$  passage in the RF cavity

**synchronous phase** (invariant)

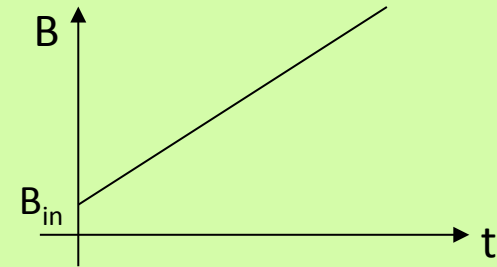
## B and $f_{RF}$ variations during acceleration

Turn-by-turn energy increase:



**B-field** increase to keep the synchronous particle on a **constant orbit**

$$\Delta W = v|_n \cdot \Delta P|_n = v|_n \cdot q\rho \Delta B|_n = v|_n \underbrace{T_{rev}|_n}_L q\rho \underbrace{\frac{\Delta B|_n}{T_{rev}|_n}}_{\cong \dot{B}}$$



$$Lq\rho\dot{B} = \Delta W = qV \cos \phi_s \Rightarrow \dot{B} = \frac{V \cos \phi_s}{L\rho} \Rightarrow \dot{P} = \frac{qV \cos \phi_s}{L}$$

B has to grow linearly with time

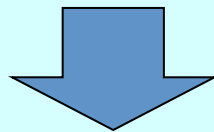
Turn-by-turn energy increase:



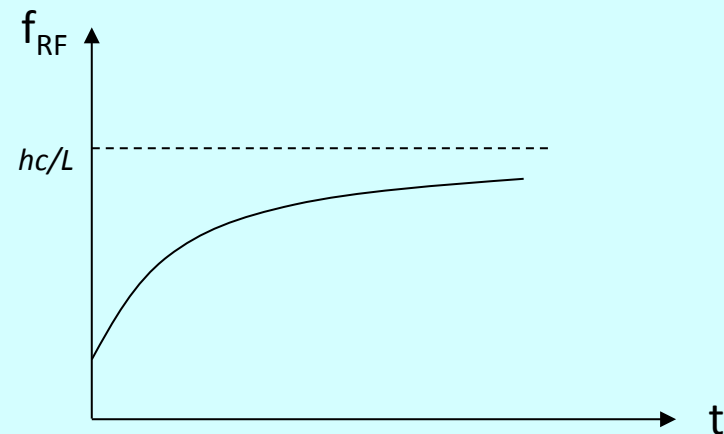
$F_{rev}$  variation



$f_{RF}$  has to be continuously changed to **keep synchronized** the accelerating field and the synchronous particle passages



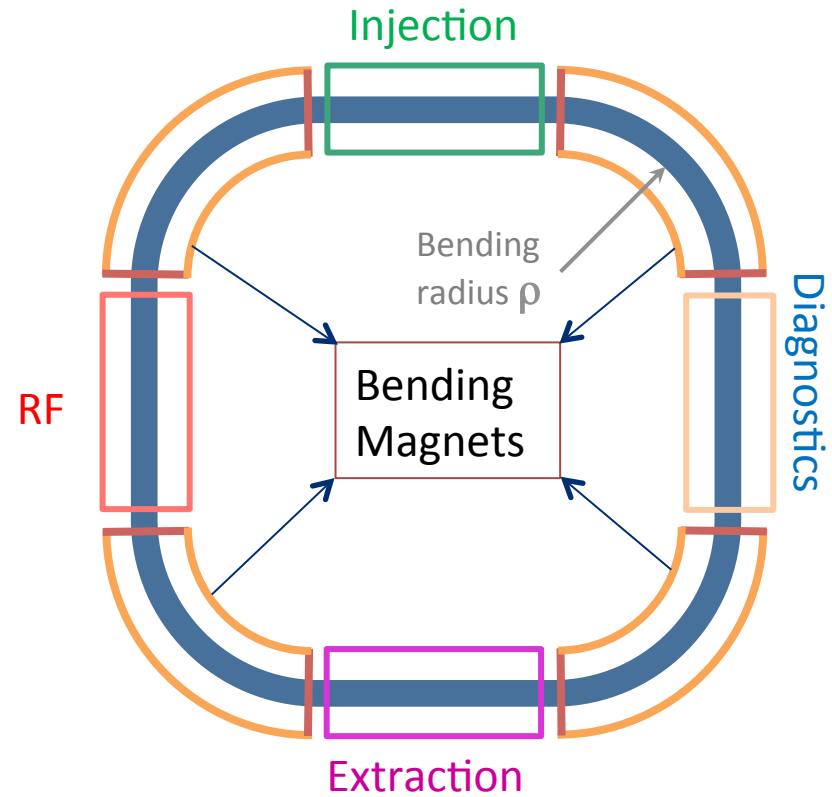
$$f_{RF}|_n = \frac{h}{L} v|_n = \frac{hc}{L} \beta|_n = \frac{hc}{L} \frac{1}{\sqrt{1 + (m_0 c / q\rho B|_n)^2}}$$



$f_{RF}$  asymptotically tends to  $hc/L$  as  $B$  becomes much larger than  $m_0 c / q\rho$ , i.e. when the particle becomes ultra-relativistic and  $v \rightarrow c$

# Exercise : let's design a synchrotron!

Particle	Protons
Rest energy	$W_0 = 938 \text{ MeV}$
Injection energy	$W_{in} = 10 \text{ MeV}$ $\gamma_{in} \approx 1.01$ $\beta_{in} \approx 0.14$
Extraction energy	$W_{out} = 400 \text{ MeV}$ $\gamma_{out} \approx 1.43$ $\beta_{out} \approx 0.71$
B field @ extraction	$B_{max} = 1.0 \text{ T}$
Bending radius	
B field @ inction	?
Total length	
RF kick Voltage	?
Magnetic field slope	?
Energy ramp duration	?
Number of turns	?
Revolution frequency	
RF frequency	



$$B = \frac{mv}{q\rho} \Rightarrow \begin{cases} \rho = \frac{W_0/q}{c} \frac{(\gamma\beta)_{out}}{B_{max}} \approx 3.2 \text{ m} & V_{RF} = \sqrt{2P_{RF}R} \approx \text{few kV} \\ B_{in} = \frac{W_0/q}{c} \frac{(\gamma\beta)_{in}}{\rho} = 0.14 \text{ T} & \dot{B} = \frac{V_{RF} \cos \phi_s}{\rho L} \approx 7.81 \text{ T/s} \end{cases}$$

$$T_{ramp} = \frac{B_{out} - B_{in}}{\dot{B}} \approx 0.11 \text{ s} \quad N_{turns} = \frac{\Delta W}{qV_{RF} \cos \phi_s} = \frac{390 \text{ MeV}}{1 \text{ kV}} \approx 3.9 \cdot 10^5$$

$$f_{rev} = \frac{\beta c}{L} \approx \begin{cases} 1.05 \text{ MHz @ inj.} \\ 5.33 \text{ MHz @ ext.} \end{cases}$$

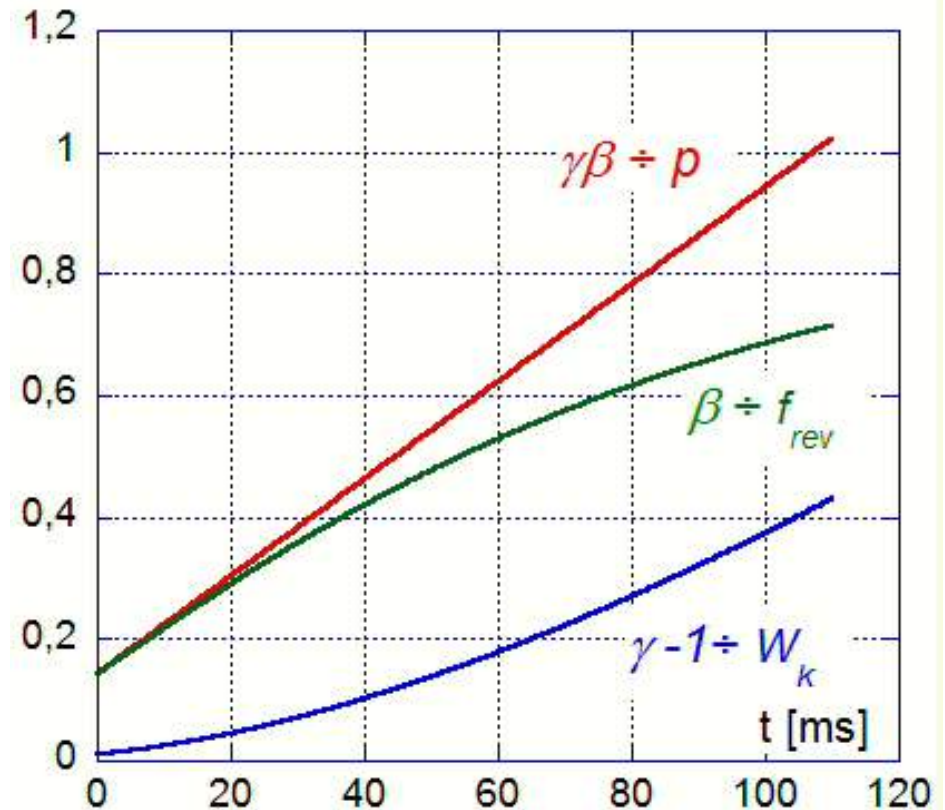
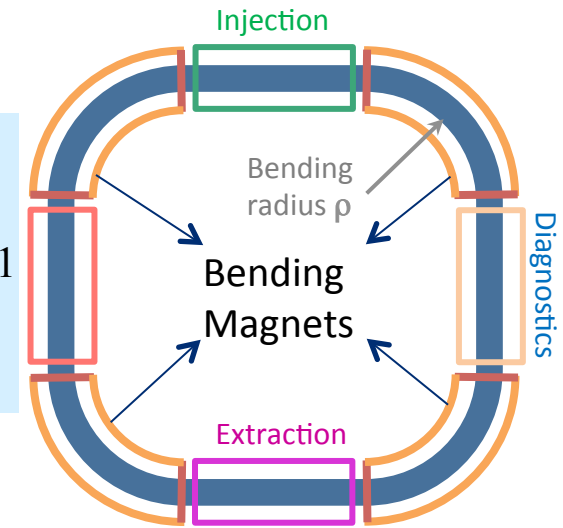
## Exercise : let's design a synchrotron!

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B field @ extraction	$B_{max} = 1.0 \text{ T}$
Bending radius	$\rho \approx 3.2 \text{ m}$
B field @ inction	$B_{in} = 0.14 \text{ T}$
Total length	$L = 40 \text{ m} (2\pi\rho + 4d)$
RF kick Voltage	$V \cdot \cos(\phi_s) \approx 1 \text{ kV}$
Magnetic field slope	$dB/dt = 7.81 \text{ T/s}$
Energy ramp duration	$T_{ramp} \approx 110 \text{ ms}$
Number of turns	$N_{turna} \approx 390000$
Revolution frequency	$f_{rev} \approx 1.7 \text{ MHz in}$ $\approx 5.0 \text{ MHz out}$
RF frequency	$f_{RF} = f_{rev} (h=1)$

$$p(t) \div \gamma(t) \cdot \beta(t) \div B_{in} + \dot{B} \cdot t$$

$$W(t) \div \gamma(t); W_k(t) \div \gamma(t) - 1$$

$$f_{rev}(t) \div \beta(t)$$





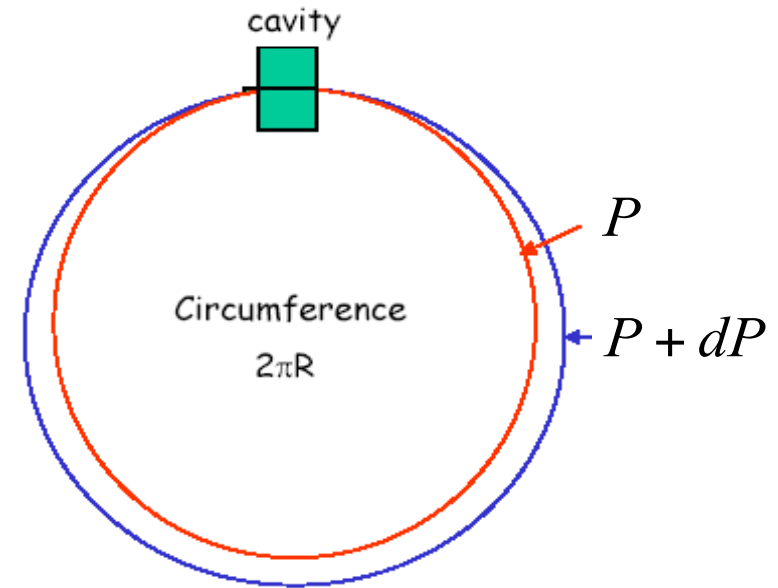
## “Momentum compaction” and transition energy

The relative orbit length variation for a given relative particle momentum variation in a circular, transversely focused accelerator is property of ring magnetic guide called “momentum compaction” factor  $\alpha$  :

$$\alpha = \frac{dL/L}{dP/P} = \frac{1}{L} \oint \frac{Disp(z)}{\rho(z)} dz$$

while the relative revolution frequency variation for a given relative particle momentum variation is called “slippage factor”  $\eta$  :

$$\eta = \frac{dF_{rev}/F_{rev}}{dP/P}$$



$$\left. \begin{aligned} F_{rev} &= \frac{\beta c}{L} \Rightarrow \frac{dF_{rev}}{F_{rev}} = \frac{d\beta}{\beta} - \frac{dL}{L} \\ \frac{dP}{P} &= \gamma^2 \frac{d\beta}{\beta} \end{aligned} \right\} \Rightarrow$$

$$\frac{dF_{rev}}{F_{rev}} = \left( \frac{1}{\gamma^2} - \alpha \right) \frac{dP}{P}$$

$$\eta = \frac{1}{\gamma^2} - \alpha$$

Transition energy is defined as:

$$\gamma_{tr} = \frac{1}{\sqrt{\alpha}}$$

Momentum compaction factors are usually positive ( $\alpha > 0$ ) in circular accelerators, so that **below transition energy** particle revolution frequency increases with energy (the velocity increase dominates the orbit length increase). On the contrary, **beyond transition energy** revolution frequency decreases while energy increases (since  $v \approx c$  and orbit length variation dominates in this case).

## Non-synchronous particles: phase-energy equations

For a **non-synchronous** particle we may define the following quantities to measure the deviation respect to the synchronous particle at the  $n^{\text{th}}$  turn:

Turn-by-turn evolution of energy and phase of non-synchronous particle is described by the following equations:



$$\begin{aligned} F_{rev}|_n &= F_{rev\_s}|_n + f_{rev}|_n \\ \phi|_n &= \phi_s|_n + \varphi|_n \\ P|_n &= P_s|_n + p|_n \\ W|_n &= W_s|_n + w|_n \end{aligned}$$

Deviations of revolution frequency, phase, energy, etc., respect to the synchronous particle:

$$\varphi|_n - \varphi|_{n-1} \cong 2\pi h F_{rev\_s}|_{n-1} T_{rev}|_{n-1} = 2\pi h F_{rev\_s}|_{n-1} \frac{1}{F_{rev}|_{n-1}} \cong -2\pi h f_{rev}|_{n-1} T_{rev\_s}|_{n-1} \Rightarrow \frac{\varphi|_n - \varphi|_{n-1}}{T_{rev\_s}|_{n-1}} = -2\pi h f_{rev}|_{n-1} \Rightarrow$$

$$\Rightarrow \frac{d\varphi}{dt} \cong -2\pi h f_{rev} \cong -\frac{2\pi h \eta F_{rev\_s}}{P_s} p \Rightarrow \frac{d\varphi}{dt} = -\frac{2\pi h \eta}{LP_s} w$$

$$w|_n - w|_{n-1} = q\hat{V}(\cos\phi_n - \cos\phi_s|_n) = q\hat{V}[\cos(\phi_s|_n + \varphi_n) - \cos\phi_s|_n] \Rightarrow \frac{w|_n - w|_{n-1}}{T_{rev\_s}} = q\hat{V}F_{rev\_s}[\cos(\phi_s|_n + \varphi|_n) - \cos\phi_s|_n] \Rightarrow$$

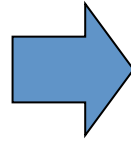
$$\Rightarrow \frac{dw}{dt} = q\hat{V}F_{rev\_s}[\cos(\phi_s + \varphi) - \cos\phi_s]$$



$$\begin{cases} \frac{d\varphi}{dt} = -\frac{2\pi h \eta}{LP_s} w \\ \frac{dw}{dt} = q\hat{V}F_{rev\_s}[\cos(\phi_s + \varphi) - \cos\phi_s] \end{cases}$$

## Synchrotron oscillations

If we consider **small oscillations** around the synchronous phase and slow variations of  $P_s$ ,  $F_{rev\_s}$  and  $\eta$ , we easily obtain the equation of an harmonic oscillator whose frequency  $f_s$  is called again **synchrotron frequency**.



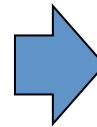
$$\frac{d^2\varphi}{dt^2} - \frac{2\pi h \eta q \hat{V} F_{rev\_s} \sin \phi_s}{LP_s} \varphi = 0$$

$$\frac{d^2\varphi}{dt^2} + (2\pi f_s)^2 \varphi = 0$$

$$f_s = \sqrt{-\frac{h \eta q \hat{V} F_{rev\_s} \sin \phi_s}{2\pi LP_s}}$$

To get **stable oscillations** the following condition have to be fulfilled:

$$\eta \sin \phi_s < 0 \Rightarrow \begin{cases} \gamma < \gamma_{tr} & -\frac{\pi}{2} < \phi_s < 0 \quad (+2n\pi) \\ \gamma > \gamma_{tr} & 0 < \phi_s < \frac{\pi}{2} \quad (+2n\pi) \end{cases}$$



If during acceleration the **transition energy is crossed**, the RF cavity phase has to be rapidly changed in order to preserve the beam longitudinal stability.

Similarly to the already shown result for the longitudinal dynamics of LINACs, particles oscillate around the synchronous one with the frequency  $f_s$  and the trajectories of their motion in the longitudinal phase space are ellipses.

However, differently from the LINAC case, the **synchrotron frequency shows a rather complex scaling law with energy**:

$$f_s \Big|_{LINAC} \div (\beta \gamma)^{-3/2} ; \quad f_s \Big|_{SINCR} \div \left| 1 - \gamma^2 / \gamma_t^2 \right|^{1/2} \gamma^{-3/2}$$

